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Estimating and testing intertemporal preferences: a unified framework for consumption, work and savings

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ABSTRACT

This dissertation contributes to the theory of intertemporal duality. A Frisch demand system derived from a consumer profit function is developed rationalizing consumption, labor supply and savings choices of households consistent with intertemporal maximization. A new functional form with many appealing properties is introduced. This functional form has the generality of rank 3 demand systems and additionally has the property that the conditioning variable of Frisch demand systems, the unobserved price of marginal utility, is solved explicitly via the inversion of the intertemporal budget constraint. This new functional from is applied to selected single headed households over a five year span using information from five waves of the Panel Study of Income Dynamics (PSID) from 1985 to1989. Household wealth from the beginning and end of this period, together with income information over this observed period is used to derive aggregate household consumption. The connection between commonly maintained primal separability restrictions and the full matrix of price cross price Frisch elasticites is demonstrated. A test of consumption-labor and time separability suggests that these restrictions ought to be rejected. Cross price Frisch elasticities are found not to equal zero and this in turn affects all estimates of consumption, labor supply and saving elasticities.

INTRODUCTION

Contemporary analyses of household expenditure patterns, those that use cross-sectional data such as consumer expenditure surveys for example, often use a dual approach to the specification of preferences. This approach does not specify a direct utility function- what is called a primal approach- where economic agents are assumed to maximize a specified utility function subject to feasibility constraints. Instead, dual approaches specify functions- for example the indirect utility function or the expenditure function- which more directly lead to demand equations which are then used to analyse observed expenditure patterns.

The use of duality as a whole manifests a keen understanding of the limitations of using a primal approach. The chief problem with the primal approach occurs as the analyst tries to derive demand equations from a specified utility function- a problem that becomes increasingly difficult as one seeks greater generality from the utility function to observe a richer and more complex range of response. The theoretical requirements of dual functions that are consistent with agent maximizing behaviour such as its degree of homogeneity, curvature and symmetry properties and so forth are well understood and are regularly applied in contemporary expenditure analyses.

Additional properties desirable in a dual specification of preferences beyond that which is consistent with agent maximizing behavior are also well understood. Flexible functional forms with sufficient generality to locally approximate the response of *any* arbitary utility function are often used. Flexible functional forms include the well known Translog model and the equally well known AIDS model. Blundell and Lewbel (1997) and

Ryan and Wales (1999) have made further generalizations in this direction with what are called rank 3 demand systems. This latest development which allows for goods to be luxury items at certain income ranges and then become inferior items at other income ranges was found to fit observed expenditure patterns better than its rank 2 predecessor.

While the theoretical development and the application of duality in these static analyses is certainly laudable, these highly general dual approaches are yet to find widespread use in the modeling of dynamic household choice. At least one reason for this lies in the data. Comprehensive information on household expenditure patterns are derived largely from cross-sectional surveys which captures information across households at one point-in-time. Such datasets cannot inform on the dynamic decision making of households.

With this as background, static demand theory has developed an understanding of the pre-conditions necessary to rationalize such an analysis. In what are known as separability arguments, the question of when an analyst may focus on goods X and goods Y and legitimately ignore goods Z is answered. Basically, one can use the apparatus of duality and analyse the consumption of X and Y without regard for Z when, in the primal utility function, goods X and Y are separable from goods Z. Thus, when using cross-sectional data for example when one has point-in-time information, one can legitimately ignore future consumption (and past consumption) if household utility is time separable. Another example occurs when household expenditures are modeled without regard to the labor earnings (or leisure consumption) of household members. In this case, an analysis of expenditure categories is legitimate if these goods are separable from the labor supply decision of household members.

The pervasive use of separable utility functions of course does not substantiate its validity. Indeed, this is often a problem of omission rather than admission. Consumer analyses may focus on the allocation of the budget across different goods and seek to estimate various elasticities for example. However in this case, the endogenieity of the budget has been ignored. This implicitly maintained assumption of a separable utility function also occurs in the labor supply literature. There exists panel datasets that have tracked household work decisions over time. These datasets are invaluable as it provides information on the dynamic labor supply choices of households. However, these panel datasets lack comprehensive data on consumption. Thus when labor supply analysts use a utility maximizing framework, labor supply dynamics are implicitly assumed to be separable from household consumption decisions. As mentioned above, the maintained assumption of separability- whether explicitly acknowledged or not- is often the result of the paucity of data available for the analysis.

The purpose of this paper is to examine how we might generalize the modeling of household behavior, especially the modeling of intertemporal household behavior and examine what insights are gained in our understanding of household behavioral responses when a more generalized approach is used. As described above, just as duality has greatly generalized static demand analysis and given analysts new insight, this paper develops intertemporal duality to generalize the intertemporal decision making of households.

The connection between separable primal preferences and the intertemporal dual is developed. As will be shown, time nonseparability of consumption implies a special relationship between present consumption and wealth which serves as the medium for the transfer of purchasing power across time. The nonseparability of labor across time implies a similar special relationship between labor and wealth. It is the use of present wealth, rather than future consumption or future labor supply which may not be available to the analyst, that enables this analysis.

This paper applies dual principles to analyse aggregate consumption, labor supply and savings choices facing households. Like static demand analysis, the equations that describe the three choices of consumption, labor supply and savings are determined in a straight forward manner rather than through an inversion that is necessary in a primal approach. The three choices are explained by equations that depend on prices and a conditioning variable that is described later. These equations are functions of what will be called own price, which is the nominal cost of the good in question, and what will be called cross prices, which is the nominal cost of some other good in question. For example, in the labor equation, own price is the price of labor or wage earned. Cross prices for the labor equation refers to the nominal price that indexes the cost of aggregate consumption or a factor that incorporates interest rates that determine the price of future goods. It will be demonstrated that the own price of real future wealth is a factor that includes an interest rate and can be treated as a price in the same way that consumption and labor each have a price. The derived equations from the dual approach that explain consumption, labor supply and wealth treats each choice in a symmetric manner. Additionally, these choices are consistent with a well posed maximization problem.

As described above, data limitations have hampered the analysis of dynamic household decision making, especially in the area of the evolution of consumption. However, what appears to be overlooked has been the availability of panel data on labor earnings *and* wealth. I derive consumption expenditure quite differently using the dynamic budget constraint. I calculate aggregate consumption expenditure by adding to beginning wealth, earned and unearned income and subtracting end wealth with adjustments for rates of return on household wealth portfolios.[1]

A new functional form with many appealing properties is introduced. This functional form draws many parallels with flexible functional forms and rank 3 demand systems used in static demand analysis. Separability restrictions are tested using this form and it conveniently amounts to whether certain estimated parameters are statistically significantly different from zero which is tested using standard statisctical methods.

The framework used in this paper draws on a comparatively lesser known dual called Frisch demand systems or lamda (λ) constant estimation (MaCurdy, 1983 and Altonji, 1986) which are derived from a dual consumer profit function (Browning, Deaton and Irish, 1985). In this framework, the unobserved price of marginal utility, $\mu \equiv 1/\lambda$, serves as a conditioning argument in the same way that utility serves as a conditioning argument in a Hicksian (or compensated) demand system and expenditure serves as a conditioning argument in a Marshallian (or uncompensated) demand system. There are advantages to using this dual over the expenditure function that gives rise to Hicksian demands or to indirect utility functions that give rise to Marshallian demands. The Frisch approach can directly test primal restrictions in the utility function avoiding the separability inflexibility described by Blackorby, Primont and Russel (1977) as will be fully explained in this paper. The Frisch system is also consistent with intertemporal optimization which will also be explained in this paper.

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[1] Portfolio, savings and wealth are used interchangeably and are nominal quantities. I use assets to denote real wealth.

Existing literature that have used consumer profit functions has been confined to comparatively simple specifications. They have not attained the generality of rank 3 Marshallian demand systems for example. One reason for this is that the resulting Frisch demand system has as a conditioning argument an unobserved price of marginal utility. The standard approach is to either difference away or treat as a fixed effect this unobserved variable. The new functional form used in this paper solves this problem by inverting the budget constraint to determine this unobservable variable. In doing this, it also solves a problem that has been gaining increasing recognition. As described by Lee (2001) the standard approach of differencing out this unobserved variable violates a classical statistical assumption of independence of explanatory variables from the error term in regression equations. Furthermore, the standard technique of using independent instruments to address this classical violation itself turns out to be problematic. This is addressed later in the paper.

This new consumer profit function is applied to test two commonly maintained, but restrictive, hypotheses; consumption-labor additivity and time separability. Although these types of separability tests seem not to have been done in the dynamic consumer context, Barnett and Hahm (1994) and the papers cited by them show its application in static producer contexts. Both restrictions are easily rejected in favor of the most general case using standard statistical arguments.

This paper next compares various estimated elastcities of the most general model to those derived by the more restrictive models in order to evaluate the impact of these maintained assumptions. I find, for example, that the Frisch (or conditional on $\mu \equiv 1/\lambda$) elasticity of consumption with respect to interest rates to be substantial in a general setting whereas this is constrained to zero under time separability. The Marshallian (or conditional on initial wealth) elasticity is related to the corresponding Frisch elasticity, so removing the restriction affects the Marshallian elasticity of consumption with respect to interest rates. The generalization reverses the conclusion one would draw from a time separable model which finds that, conditional on initial wealth, the effect of higher short term interest rates significantly reduce consumption to one where it increases consumption for wealthy households and reduces consumption for poorer households. Another result is the finding that interest rates, conditional on μ, have a significant impact on labor supply which is constrained to zero in time separable models. Because homogeneity of the profit function implies certain adding up properties in the matrix of price cross-price elasticities, removing constraints on off-diagonal Frisch elasticities changes the entire matrix of price cross-price Frisch and Marshallian elasticities.

My procedure is also able to discern how changes in wages and interest rates change μ depending on whether this is evaluated in cross section or in time series. I argue that when the response of μ to wage increases is evaluated in cross section, this is best thought of as the effect of a transitory one year increase in wages. On the other hand, when this is evaluated in time series, this can best be though of as a perturbation to the evolutionary path of wages which has some persistence. By comparing the two, I find that the wage effect on μ in time series is over 4 times stronger than it is for a transitory wage increase, suggesting if wages revert to a mean geometrically, it does so at a little over 20% annually. Conditional on initial wealth, labor supply switches from inelastic but positively sloped in response to short term wage increases to one that is backward bending for long term wage increases due to a wealth effect.

Generalizing the modeling of intertemporal household choice also changes the estimates of several other important intertemporal parameters. For example, restrictive models estimate that the rate of time preference is around 7% while in the more general model it is 1%. Additionally, intertemporal optimization that gives rise to the Euler equation implies that the elasticity of λ with respect to interest rates should be approximately one. The restricted models find estimates of this elasticity ranging from 0.317 to 0.424 whereas the preferred general model finds an estimate of 0.924. It appears that interest rates have two distinct channels of operation; one through the structural equations conditional on μ or λ and another through the Euler equation, but time separable models cannot distinguish the two separate effects.

This research contributes to the modeling of household intertemporal choice in a number of ways. This is the first time that the profit function has been applied to the full set of dynamic choices faced by households. I use a "hard" budget constraint- where initial wealth is predetermined or weakly exogenous. This contrasts with static analyses of consumer demand where total expenditure is treated as if it were exogenous when in fact it is an endogenous intertemporal choice. This arises in static analyses because a savings choice is not explicitly modeled and is justified as an analysis of temporal expenditure after an intertemporal optimization in what is presumed a two-stage budgeting process.

In one sense, I have simplified the problem. I do not think of intertemporal optimizations as maximization of utility functions that are somehow defined over vague and distant future commodity bundles. Instead, I look at the immediate choice facing households: How much do I consume, work and save today? The profit function treats each of these choice variables in a symmetric and mutually consistent manner. Further, my profit function is globally regular and derived from an approximation scheme with desirable properties. Most importantly perhaps, I show that models with separability restrictions which inform most of our current understanding of labor, consumption, savings and intertemporal elasticities should be rejected in favor of a general model when modeling household intertemporal choice.

The following section presents background followed by a theoretical discussion of my approach and the new functional form. This is followed by a section describing the data used for this analysis. This is followed by a discussion of my results and a conclusion where I emphasize the possible extensions to this basic framework.

BACKGROUND

This section highlight reviews of the literature on consumption, labor supply and savings. Much of the analysis in this area arises from a primal specification of preferences where authors have been confined to simple specification of preferences for various reasons. I also review dual specifications of preferences. One goal of the review is to note the simpler structural equations typically used to fit the data so as to highlight the generality and symmetry I attain.

Consumer theory is surveyed in Blundell (1988) where issues of separability, additivity and preference restriction are well addressed. As discussed in this article, time-separability of the utility is the only justification for basing present choice variables on present prices. Without this crucial assumption, present choices will be based on prices of other periods which will typically exceed the data available to the econometrician. For example, models of labor supply (or consumption) require the future path of wages (and prices).

Another survey of consumption theory is by Elmendorf (1996). Here the focus is on how interest rates can affect consumption, and via this channel, affect savings. Three interest rate mechanisms are identified. The first is the substitution effect that can be seen in a simple model where utility is defined over consumption over two periods: the present and the future period. An increase in the interest rate makes present consumption more expensive relative to future consumption. Consequently, a rational consumer substitutes future consumption for the comparatively more expensive present consumption, an effect that unequivocally leads to lower present consumption with higher interest rates. A second mechanism is that an increase in interest rates lowers the present discounted value of future consumption. Higher interest rates imply fewer present dollars are required to finance a given level of consumption, an income effect that leads unequivocally from higher interest rates to higher present consumption if present consumption is normal. A third effect is that higher interest rates lead to a fall in the present value of future income. This future income may be earned income. It can also affect the present value of income from financial investments and the impact of interest rate changes may or may not be immediately capitalized in the value of the financial asset. The overall balance of these three mechanisms on how interest rates affect either present or future consumption is ambiguous. Other surveys of consumption theory include Deaton (1992) where nearly all specifications of utility are primal and Barnett, Fisher and Serletis (1992) where the focus in on placing monetary aggregates in the utility function.

Surveys of labor supply include Blundell and MaCurdy (1999), Pencavel (1986) and Killingsworth and Heckman (1986) where λ-constant estimation, the Frisch demand system and the consumer profit function are discussed. The Frisch approach to estimation is conditional on a constant marginal utility in the same way a Hicksian approach is conditional on a constant utility and Marshallian approach is conditional on a constant expenditure. The former approach has been adopted by several authors in the labor supply literature but has not been used in the consumption literature except where both consumption and labor (or leisure) are studied jointly. Several studies have focused specifically on the non-separability of consumption and labor which is a generalization over models studying consumption or labor in isolation but nearly all studies maintain time separability because to make this further generalization requires, using their approach, future prices which then requires panel data.

A survey of household savings can be found in Browning and Lusardi (1996), however as they point out, theories of savings are predominately theories of intertemporal and life-cycle consumption. As they show, except for buffer-stock motives for holding wealth, where households are mindful of possible future income shocks and hold a reserve of funds to smooth future consumption, there is little positive theory in the area of wealth or savings.

The closest article to my present work is Browning, Deaton and Irish (1985, BDI) and the extension by Merrigan (1994). The former estimates labor supply for males and consumption using the Frisch structural equations;

$h = \alpha_1 + \beta_1 \ln w + \theta_1 \sqrt{p/w} - \beta_1 \ln \mu$ and,

$c = \alpha_2 + \beta_2 \ln w + \theta_2 \sqrt{w/p} - \beta_2 \ln \mu$

for labor supply, h, and consumption, c, respectively. How equations such as these are derived is shown later as structural equations are derived for my model. These structural equations depend on wages, w, prices, p and price of marginal utility, μ and are linear in parameters which aid estimation. These equations are also linear in the log of μ. Since μ is unobservable, their strategy for the estimation of the parameters is to difference these equations to cancel out $\ln \mu$ based on assumptions of how μ in one period is related to μ in the next. This, together with an inherent problem with this approach, is discussed a little later.

A total of 6 parameters are estimated by BDI and a test of the symmetry condition, $\theta_1 = \theta_2$, is performed. These structural equations were fitted to the mean labor supply and consumption levels of cohorts from the British Family Expenditure Surveys (BFES) from

1970-1977. Since the BFES is not a true panel, but instead a sequence of cross-sectional expenditure surveys, a synthetic cohort is constructed from age groupings followed across the 7 years and their model seeks to fit the means of each group. Needless to say, cohort heterogeneity is ignored in this analysis as is income uncertainty at the individual level. They do however allow for economy wide shocks in their uncertainty model.

The extension by Merrigan (1994) uses 103 observations from the PSID and follows the household over 13 consecutive years. He fits the same structural equations as BDI but over 3 goods: the husband's labor supply, the wife's labor supply and household consumption. Like BDI, his method of solving for unobserved μ is to use a structural equation which is additive in log μ and then to difference it out by fitting structural equations to changes in leisure demand and changes in consumption. Both of these studies, as is typical of the literature as a whole, report wide-ranging temporal and intertemporal elasticity estimates.

In an important paper that is likely to shift the ground for future labor supply studies, Lee (2001) explains the reasons why labor studies have found such wide-ranging elasticity estimates and wide standard errors on these estimates. The problem is inherent with two-stage instrumental variable estimation and is now being increasingly recognized. It is necessary to use instruments to wages, rather than wages itself, in the regression of hours to wages.

To use wages directly results in a violation of a key classical regression assumption of independence of the error term from the explanatory variables (wages). The correlation between the error term and the explanatory variable leads to bias in estimated parameters. To circumvent this problem, one may use instruments to wages rather than wages itself if the instruments are independent of the error term. This leads to an unbiased parameter estimates. However, as Lee points out, this is an asymptotic result and because the instruments for the wage regression are typically weak, biases are not eliminated by the sample sizes typical of labor supply studies.

To see the violation of the classical regression assumption, the structural equation for the log of labor supply written in its most general form with an additive $\log \mu$ can be written as

$$\ln h = f(p, Z) + \alpha \ln \mu$$

where h is labor supply, p are prices including wage rates and Z are demographic conditioning variables. To remove log μ, one models changes in μ from one period to the next according to a priori assumptions about the behavior of economic agents. Naturally, because μ cannot be observed, these assumptions cannot be tested directly. However, assuming economic theory can inform on the behavior of rational agents, the structural equation for two consecutive periods can be differenced in such a way as to have the unobserved term drop out.

For example, it might be assumed that an agent seeks to maximize his welfare across time. This implies that the agent will seek to transfer purchasing power intertemporally until the discounted marginal utility of future wealth is equal to the present marginal utility of wealth. If an agent is better off spending a dollar today than saving the dollar for consumption the next period, then the agent has not optimized his or her decision at the intertemporal margin. This intertemporal optimality condition is the Euler equation, $\delta E\lambda_{t+1} = (1 + i_t)^{-1}\lambda_t$ where δ is a time discount factor, E is an expectations operator, i is the interest rate, $\lambda \equiv 1/\mu$ is the marginal utility of income (or the inverse the price of marginal utility) and the subscripts denote time.

While the Euler equation equates *expected* future marginal utility with discounted present marginal utility, *actual* future utility may differ from expected future utility depending on the realization of economic variables. For example, an unexpected increase in wages will increase the welfare of workers and decrease the marginal utility of income. In this case, realized λ will be less than $E\lambda$. In this way, error between actual and expected marginal utility and wages may be correlated.

The set of economic variables which are relevant and in the information set of economic agents and the process by which these random variables are realized are crucial economic assumptions about the agent's economic environment. Let the relationship between actual discounted future marginal utility be related to present marginal utility by a multiplicative error term, ε. This can be expressed as $\lambda_{t+1} = \delta^{-1}(1+i_t)^{-1}\lambda_t\varepsilon_{t+1}$. The Euler equation implies that $E_t(\varepsilon_{t+1}) = 1$, an equation that assumes that agents do not make systematic errors in making their forecasts. This equation in log form is $\ln\lambda_{t+1} = \ln\lambda_t - \ln\delta(1+i_t) + \ln\varepsilon_{t+1}$ and can be the means by which structural equations additive in $\log\mu$ might be differenced.

However, labor supply in period t+1 will have both t+1 wages and error. The forecast error will include how realized wages in t+1 differ from expected t+1 wages and the error term is not independent of wages in the next period. Thus, although one can use the log form of the Euler equation and difference two consecutive labor supply equations, the error term, being correlated with t+1 wages, will violate classical regression assumptions leading to biased estimates. This is why it is necessary for modelers to instrumented wages. It is assumed that the instruments are independent of the error term.

However, as Lee (2001) points out, instrumental variable estimation in finite samples can be severely biased. When the instrument set for wages is particularly weak which is often the case, it leads to open-ended $(-\infty, \infty)$ robust confidence intervals and an uninformative estimate. Clearly, the unobservability of the marginal utility of income, λ, poses substantial challenges for the modeling of intertemporal choice.

My innovation and contribution to this literature starts with the invention of a new functional form which allows for an implicitly defined λ in a budget constraint to be inverted so as to have an explicit expression. The explicit expression for λ can then be substituted into Frisch structural equations. This approach overcomes Lee's weak instrument problem. This approach also allows for the estimation of parameters to the structural equations that are based solely on exogenous (or predetermined) variables. Additionally, my functional form allows for comparatively greater generality of the structural equations. I turn to a description of the model and approach next.

ECONOMIC MODEL

In this section, I present a brief primer on Frisch demand systems and present the intertemporal decision as a one year problem where households evaluate prices, wages and interest rates together with an initial level of wealth to plan their consumption, labor and savings choices. I develop the connection between a household's present value of future consumption and a contemporaneous interest rate which I call an interest factor. I discuss the profit function which incorporates my interest factor as well as theoretical features of my model. As will be seen next in the data section, the data available do not include information on beginning-of-year and end-of-year wealth but rather wealth at the beginning and end of a five-year span. The issue of matching this model with 5 years of incomplete data is addressed in the data section but it suffices here to state that this is considered as a sequence of 5 one-year optimizations with household re-optimizing with the realization of new wage and interest information each year.

To introduce the Frisch demand system, consider the log form of a Cobb-Douglas utility function over x, $U(x) = \sum \alpha_i \ln x_i$ where x_i denotes good i in the utility function. The budget constraint that limits attainable utility is $\sum p_i x_i = m$ where p_i denotes the price of good i and m is total expenditure on goods. The maximization problem faced by the economic agent is to maximize U(x) over choice variables x subject to the budget constraint $\sum p_i x_i = m$. The Lagrangian for this problem is

$L(x, \lambda, p, m) = U(x) + \lambda(m - \sum p_i x_i)$ with first order conditions

$\dfrac{dL(x, \lambda, p, m)}{dx_i} = \dfrac{dU(x)}{dx_i} - \lambda p_i = 0$ and

$$\frac{dL(x,\lambda,p,m)}{d\lambda} = m - \sum p_i x_i = 0.$$

The first order conditions of the Lagrangian using the log form of the Cobb-Douglas utility function is comparatively simple giving $\alpha_i / x_i - \lambda p_i = 0$. However, one often want to obtain a solution to this expressing demand for good i in terms of prices and expenditure, $x_i = x_i(p,m)$. This is called the Marshallian demand for x (which incidentally has observable arguments).

In the simple log Cobb-Douglas case, one can re-arrange the first order conditions to obtain $x_i = \alpha_i / \lambda p_i$, a process known as inversion of the first order conditions. This expression is a Frisch demand equation which expresses demands as a function of parameters, prices and the marginal utility of income (or its inverse). Finally, to obtain a Marshallian demand equation one must solve for $\lambda = \lambda(p,m,\alpha)$. This term is implicitly defined in the budget equation as the Frisch demand equations are substituted into the budget constraint. To continue with this example of the log Cobb-Douglas utility function, substituting Frisch demand equation in the budget constraint gives,

$\sum p_i x_i(\alpha,p,\lambda) = \sum p_i(\alpha_i / \lambda p_i) = \frac{1}{\lambda}\sum \alpha_i = m$. To solve for λ (or μ), one can re-arrange to give the price of utility, $\mu = \frac{1}{\lambda} = \frac{m}{\sum \alpha_i}$. This last step is called the inversion of the budget constraint. Substituting this into the Frisch demand, $x_i = \alpha_i / \lambda p_i$, one obtains the Marshallian demand for x, $x_i = \frac{m}{p_i}\frac{\alpha_i}{\sum \alpha_i}$. For future reference, note that the ratio $\frac{\alpha_i}{\sum \alpha_i}$ is an expression of apportionment or share of expenditure towards good i. To see this, note that $p_i x_i = m \frac{\alpha_i}{\sum \alpha_i}$. As is well known, Cobb-Douglas utility leads to expenditure that is proportional to income. Note also that total expenditure is exhausted on the expenditure of all goods. As will be seen later, a similar apportionment will be shown later in the function I employ. My function will have an apportionment proportional to λ and another apportionment proportional to μ.

To summarize this primer, the first order condition equates the marginal utility of consuming good i with the price of good i and λ. While the inversion of these first order conditions was easy in the log form Cobb-Douglas, this is not generally true because marginal utility of consuming good i may be a function of all goods, not just of good i. This difficulty has lead to the use of duality which avoids the problem of inverting the first order conditions. This is discussed in more detail later. Leaving this issue aside for now and continuing with this summary, the inverted first order conditions lead to Frisch demand equations which are conditional on λ. Ordinarily, one then solves for λ in terms of prices, incomes and parameters using the budget constraint. Note that in this set-up, total expenditure m is exhausted on all the goods under consideration. The corollary is if a good is missing, then one could not invert and solve for λ. In this paper however, the intertemporal nature of the budget constraint and the limitations in data need careful consideration. This is developed next.

Consider a common specification of the intertemporal budget constraint: $W_t + y_t + w_t h_t - p_t c_t - (1+i_t)^{-1} W_{t+1} = 0$ where W is the nominal value of wealth, y is tax adjusted unearned income, w is after-tax nominal wage, h is hours of work, p is consumer prices, c is consumption, i is nominal interest and subscript t denotes time. The choice variables are c_t,

h_t and W_{t+1}. There are real and nominal variables in this specification. Define the real wealth variable $A_t = W_t/p_t$. The budget constraint can now be written in terms of real choice variables: $W_t + y_t + w_t h_t - p_t c_t - r_t A_{t+1} = 0$ where $r_t = p_{t+1}/(1+i_t)$ is the present nominal price of future real consumption and wealth. I call r_t the interest factor. This is a nominal quantity because dividing this by p_t gives a *real* time t cost of next period consumption. Dividing the LHS of the intertemporal budget constraint by p_t expresses the budget constraint exclusively in terms of real prices and quantities. The constraint, written in this form, alerts us to the fact that the three real choice variables c_t, h_t and A_{t+1} are linear in prices p_t, wages w_t, and interest factor r_t and that predetermined nominal wealth W_t and exogenous income y_t has the effect of shifting the level at which the constraint binds. Clearly then, p_t, w_t and r_t are the correct prices corresponding to consumption, labor supply and future real wealth.

Intertemporal maximization, what is called the primal problem, is often modeled with a recursive value function. Consider the value function

$$\begin{aligned} &V(W_t + y_t, p_t, w_t, r_t : c_{t-1}, h_{t-1}) = \\ &\max_{c_t, h_t, A_{t+1}} \left[U(c_t, c_{t-1}, h_t, h_{t-1}) + E_t \delta V(p_{t+1} A_{t+1} + y_{t+1}, p_{t+1}, w_{t+1}, r_{t+1} : c_t, h_t) \right] \qquad (1) \\ &\text{s.t.} \quad W_t + y_t + w_t h_t - p_t c_t - r_t A_{t+1} = 0 \end{aligned}$$

where V is the value function, U is the temporal utility function, E is the expectations operator with respect to future dated information and δ is the time discount factor. Meghir and Weber (1996) use a value function similar to this but without labor supply which of course assumes consumption labor separability. The appearance of past consumption in the utility function allows for durability in the case of $U_{c(t)c(t-1)} < 0$ and for habits in the case of

$U_{c(t)c(t-1)} > 0$.[2] A symmetric time dependence is allowed for in work hours allowing the disutility of work to change with past work experience. In the case of durability or habit formation, the utility function is not time-separable and present consumption and work will affect future utility.

Since one is not concerned with the simultaneous choice of both hours at work and time devoted to leisure, one can internalize the household's time constraint simply by $h_t = T - l_t$ where T is the time endowment and l_t is leisure time.

The Lagrangian for the primal problem is

$$\begin{aligned} L &= U(c_t, c_{t-1}, h_t, h_{t-1}) + E_t\delta V(p_t A_{t+1} + y_{t+1}, p_{t+1}, w_{t+1}, r_{t+1} : c_t, h_t) + \\ &\lambda_t(W_t + y_t + w_t h_t - p_t c_t - r_t A_{t+1}) \end{aligned} \tag{2}$$

with first order conditions over present choice variables,

$$\begin{aligned} L_c &= U_c + E_t\delta V_c - \lambda p = 0 \\ L_h &= U_h + E_t\delta V_h + \lambda w = 0 \\ L_A &= E_t\delta V_A - \lambda r = 0 \end{aligned} \tag{3}$$

where time subscripts are removed for now. The first order condition (3) holds as an identity, irrespective of the prices, wages and interest rates that prevail since the economic agent is assumed to be optimizing in the face of any price.

The total differentiation of the first of the first order conditions with respect to p gives,

$L_{cp} = (U_{cc} + E_t\delta V_{cc})\frac{dc}{dp} + (U_{ch} + E_t\delta V_{ch})\frac{dh}{dp} + (U_{cA} + E_t\delta V_{cA})\frac{dA}{dp} - \lambda = 0$ and the total differentiation of the first of the first order conditions with respect to w gives,

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[2] Throughout this paper, the subscripting of a function with an argument shall denote differentiation with respect to the argument. Subscripts will also be used to denote time and indexing variables. Where double

$L_{cw} = (U_{cc} + E_t\delta V_{cc})\frac{dc}{dw} + (U_{ch} + E_t\delta V_{ch})\frac{dh}{dw} + (U_{cA} + E_t\delta V_{cA})\frac{dA}{dw} = 0$. In turn the first of the first order conditions can be totally differentiated with respect to r. Then the second of the first order conditions can be differentiated with respect to the 3 prices, p, w and r. The third of the first order conditions can also be differentiated with respect to 3 prices, p, w and r. The total differentiation of the 3 first order conditions (3) with respect to 3 prices; p, w and r gives the 9 equations in matrix form,[3]

$$\begin{bmatrix} L_{cc} & L_{ch} & L_{cA} \\ L_{ch} & L_{hh} & L_{hA} \\ L_{cA} & L_{hA} & L_{AA} \end{bmatrix} \begin{bmatrix} c_p & c_w & c_r \\ h_p & h_w & h_r \\ A_p & A_w & A_r \end{bmatrix} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \tag{4}$$

where $L_{cc} = (U_{cc} + E_t\delta V_{cc})$, $L_{ch} = (U_{ch} + E_t\delta V_{ch})$, $L_{cA} = (U_{cA} + E_t\delta V_{cA})$ and so on.

Let **M** denote the first matrix on the LHS and **N** denote the second matrix. In the most restrictive case where utility, $U = U^c(c_t) + U^h(h_t)$, is additive, the value function does not have prior consumption or work hours. Additionally, the cross derivative $U_{ch} = 0$. This means that **M** is a diagonal matrix which implies **N** is also diagonal. The zeros in the off-diagonal elements of **N** implies that the structural equations for consumption, work and wealth are functions of own price only. The inversion of the first order conditions (3) give the consumption function, $c_t^* = f^A(\lambda_t p_t)$, the labor supply function, $h_t^* = g^A(\lambda_t w_t)$, and the wealth or savings function, $W_{t+1}^* = h(\lambda_t r_t)$[4] where the asterisk denotes model predicted quantities.

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subscripting is required as above, for legibility, Iuse parentheses for the second subscripted variable.

[3] Although there are 9 equations, the leftmost matrix on the LHS is symmetric by Young's Theorem which implies 6 independent equations.

[4] I have ignored the mechanism whereby w_t and r_t condition expectations of future w_{t+j} and r_{t+j}, j>0. This is modeled later in the paper.

Relaxing additivity, the time separable utility function $U = U^{TS}(c_t, h_t)$ has first order conditions $U_c^{TS}(c_t, h_t) = \lambda_t p_t$ and $U_h^{TS}(c_t, h_t) = -\lambda_t w_t$. This adds a non-zero element L_{ch} to matrix **M** and implies the corresponding non-zero elements in matrix **N**. Inversion now leads to the consumption function $c_t^* = f^{TS}(\lambda_t p_t, \lambda_t w_t)$ and the labor supply function $h_t^* = g^{TS}(\lambda_t p_t, \lambda_t w_t)$. Wages now appear in the consumption function and prices appear in the labor supply function.

In the most general case where consumption-labor additivity and time separability are relaxed, the appearance of prior consumption and labor supply in the value function implies that none of the off-diagonal elements of **M** are zero which implies none of the off-diagonal elements of **N** are zero. Thus the most general case will have a system of structural equations $c_t^* = f^G(\lambda_t p_t, \lambda_t w_t, \lambda_t r_t)$, $h_t^* = g^G(\lambda_t p_t, \lambda_t w_t, \lambda_t r_t)$ and $W_{t+1}^* = h^G(\lambda_t p_t, \lambda_t w_t, \lambda_t r_t,)$ where all prices enter into the each equation. The objective then is to develop a utility consistent system of equations for consumption, labor and savings and examine the significance of these cross-price terms.

To test this, I use duality theory (Diewert, 1974) rather than the specification of a particular utility function in a primal approach to identify preferences. Duality techniques specify a parent function that spawns the structural equations via differentiation with respect to a choice variable's price. In the production or firm context where the profit function is widely used, this is known as Hotelling's Lemma. A similar derivative property exists for the cost function which is known as Sheppard's Lemma. Both are known as derivative properties and result from the envelope theorem. To see this, let f=f(x,p) where f is an objective function, x are choice variables and p are environmental (or exogenous) variables. Let us assume the objective is the maximization of f. (There is no loss of generality here. If the objective f was a minimization problem, then one may define g = -f and proceed with the maximization of g over choice variables x and environmental variables p.) If f(x,p) is maximized over choice variables x for any variable p, then x will be a function of p, i.e., x=x(p). Define the function g(p)=f(x(p),p). The envelope theorem states that;

$\frac{dg}{dp} = \frac{\partial f}{\partial x}\frac{dx}{dp} + \frac{\partial f}{\partial p} = \frac{\partial f}{\partial p}$ since $\frac{\partial f}{\partial x} = 0$ in the maximization. In words, the envelope theorem states that the derivative of the maximized function g with respect to p is equal to the "direct effect," the partial derivative of f with respect to p, and that one can ignore the "indirect effect" which operates through the choice variables because the choice variables will be optimized and the objective function will be stationary with respect to it.

The dual profit function associated with the value function (1) is

$$\pi(p_t, w_t, r_t, \mu_t) = \max_{c_t, h_t, A_{t+1}} \begin{bmatrix} \mu_t U(c_t, c_{t-1}, h_t, h_{t-1}) + \mu_t E_t \delta V(W_{t+1} + y_{t+1}, p_{t+1}, w_{t+1}, r_{t+1} : c_t, h_t) \\ + w_t h_t - p_t c_t - r_t A_{t+1} \end{bmatrix} \tag{5}$$

where μ_t is the inverse of the time-t Lagrangian multiplier of the constraint or the price of utility. Note that the first order conditions that stem from (5) are identical to the first order conditions of (1). To see this, note that the first order conditions of (5) are;

$$\mu_t U_{c(t)} + \mu_t E_t \delta V_{c(t)} - p_t = 0$$
$$\mu_t U_{h(t)} + \mu_t E_t \delta V_{h(t)} + w_t = 0$$
$$\mu_t E_t \delta V_{A(t)} - r_t = 0$$

which, if one multiplies by $\lambda_t \equiv 1/\mu_t$ are identical to the first order conditions of (3).

This derivative property mentioned above is a considerable convenience and circumvents the problem of inverting the first order conditions (3) to find a solution. The

Frisch structural equations for consumption, labor supply and savings are obtained simply by taking the derivative of the profit function with respect to own price. To see this, note that the choice variables c_t, h_t, A_{t+1} will be functions of p_t, w_t, r_t and μ_t. The derivative of π in (5) with respect to p_t for example will be,

$$\frac{d\pi}{dp_t} = \left(\mu_t \frac{dU}{dc_t} + \mu_t E_t \delta \frac{dV}{dc_t} - p_t\right)\frac{dc_t}{dp_t} + \left(\mu_t \frac{dU}{dh_t} + \mu_t E_t \delta \frac{dV}{dh_t} + w_t\right)\frac{dh_t}{dp_t} +$$

$$\left(\mu_t E_t \delta \frac{dV}{dA_t} - r_t\right)\frac{dA_t}{dp_t} - c_t = -c_t,$$

model estimated consumption, -c*.[5] The expression the parentheses are equal to zero because of the first order conditions to (2) or (5) mentioned already. Similarly the direct effect of the derivative of the profit function with respects to w isolates h and the direct effect of the derivative of the profit function with respect to r isolates A.[6] There is another advantage of the dual approach related to the modeling of the evolution of λ which I discuss more fully in the data section.

Recognizing data limitations that lie ahead, prior consumption and work and future prices are subsumed in the profit function in accordance with a general approach of determining current choice variables from available exogenous variables.

Two important features of the dual profit function (5) are the unobserved variable μ_t and the expectation over the following period's value function. Regarding the unobserved μ_t, clearly additional structure is needed for the empirical implementation of the profit function for which I will use the ex-post budget constraint and assumptions about its

[5] Time subscripts are henceforth dropped unless needed for exposition.
[6] Note that consumption and assets are given by the negative of the derivative while labor hours are given by the positive.

evolution. This is covered in detail later. Regarding the future value of the value function, note that expectations are determined conditional on information available at time t. While it is impossible for the econometrician to know exactly what is in an agent's information set, it would seem reasonable that it includes contemporaneous values of p_t, w_t and r_t.

The theory of consumer profit functions are covered by Deaton, Browning and Irish (1985), Kim (1993), Chaudhuri (1995, 1996) and McLaughlin (1995). A profit function is said to be regular if it is homogeneous of degree one and convex in its arguments. A function, f, is homogeneous of degree one if $f(\zeta x) = \zeta f(x)$ for ζ greater than zero. It is clear from inspection of (5) that any positive constant multiplying prices and μ can be factored out of the maximization of (5). A second necessary property of a profit function is convexity. A function, f, is convex if all points on the linear interpolation between $f(p^1)$ and $f(p^2)$ lie above $f(p^v)$ where p^v lies between p^1 and p^2, i.e. $vf(p^1) + (1-v)f(p^2) \geq f(p^v)$, $p^v = vp^1 + (1-v)p^2$, $v \in [0,1]$. The profit function is convex in its arguments and this arises because of agent optimization. To see this, let the expression within the parentheses of (5) be represented by f(x,p) where x are the choice variables and p the arguments of the profit function. To maximize the function f, the choice variables x will generally be a function of p so, alternatively, one may write (5) as $\pi(p) = f(x(p), p)$. Now consider the profit function evaluated at any point between p^1 and p^2, $\pi(p^v) = f(x(p^v), p^v)$. Because of the linearity of the profit function in arguments p, one has the inequality

$$\pi(p^v) = vf(x(p^v), p^1) + (1-v)f(x(p^v), p^2) \leq$$
$$vf(x(p^1), p^1) + (1-v)f(x(p^2), p^2) = v\pi(p^1) + (1-v)\pi(p^2).$$

Thus, a chosen functional form of the profit function which is not homogeneous of degree one and convex is inconsistent with agent maximization assumed in (5).

A desirable property of any dual functional form is flexibility. Consider any arbitrary utility function evaluated at a certain point in n-goods space. The first derivative of the utility function at this point lead to n gradients and the matrix of second derivatives of the utility function contained in the Hessian lead to curvatures in n(n+1)/2 directions. Since an affine transformation of the utility function is innocuous because it is an equally valid representation of preferences, what is material is the (n-1) relative gradients and the (n(n+1)/2 -1) relative curvatures. A functional form with sufficient parameters to independently estimate each of these relative gradients and curvatures of an arbitrary utility function is said to be flexible.[7]

One of the challenges of taking a new approach to the data is often developing a parametric specification to fully rationalize the data. In addition to the homogeneity and convexity of the profit function in the observable price variables, consideration needs to be given for the unobserved μ. I develop a function that is not only globally convex in prices and in unobserved μ, it lends itself to an explicit expression of μ on inversion of the dynamic budget constraint. As far as I am aware, this is a new functional form and the only one I know of which is flexible and allows an explicit expression for μ on the inversion of the budget constraint. The profit function (5) which represents a household's maximization problem is parameterized by

$$\pi(p, w, r, \mu : z) = \pi^{\alpha}(P, \mu, \alpha)\mu + \pi^{\beta}(P, \beta : z) + \pi^{\gamma}(P, \gamma) / \mu \qquad (6)$$

[7] Flexible in my paper does not mean minimally flexible, another usage, which is a functional form with the minimum number of parameters required for flexibility.

where $P = (p_1, p_2, p_3)' = (p, w, r)'$ is a vector of prices, z=(age, age^2, age^3, number of dependents, sex of household head)' is a vector of demographic characteristics and α, β and γ are vectors of parameters to be estimated. The sub-functions $\pi^{\alpha}(P, \mu, \alpha)$, $\pi^{\beta}(P, \beta : z)$ and $\pi^{\gamma}(P, \gamma)$ are given by

$$\pi^{\alpha}(P, \mu, \alpha) = \sum_{i=1}^{3} \alpha_{ii} \ln(\mu / p_i) + \sum_{i=1}^{3} \sum_{j>i} \alpha_{ij} \ln(\mu / (p_i + \alpha_{ijj} p_j)) \tag{7}$$

$$\pi^{\beta}(P, \beta : z) =$$
$$\sum_{i=1}^{2} d_i [\beta_{i10} p_1 + \sum_{j=2}^{3} p_j (\beta_{ij0} + \beta_{ij1} \text{age} + \beta_{ij2} \text{age}^2 + \beta_{ij3} \text{age}^3 + \beta_{ij4} \text{dependents})] \tag{8}$$

where d_i is the indicator for the sex of the household head, i=1 indicating male and i=2 indicating female[8], and

$$\pi^{\gamma}(P, \gamma) = [(\gamma_{11} p + \gamma_{12} w + \gamma_{13} r)^2 + (\gamma_{22} w + \gamma_{23} r)^2 + (\gamma_{33} r)^2] / 2 \tag{9}$$

This profit function is linearly homogeneous as is required to represent household maximization. Sub-function π^{α} is homogeneous of degree zero as can be seen from (7) since any positive constant on the numerator will cancel that on the denominator. As sub-function π^{α} multiplies μ in (6), this component of the profit function is homogeneous of degree one. Sub-function π^{β} is homogeneous of degree one from (8) as prices enter linearly. Finally, from (9), it can be seen that sub-function π^{γ} is homogeneous of degree two as it is a quadratic form. As sub-function π^{γ} is divided by μ, this component of the profit function is also homogeneous of degree one. Thus, the composite profit function is homogeneous of degree one as required.

Additionally, the profit function is globally convex for $\alpha_{ij} \geq 0$ and $\alpha_{iij} \geq 0$. This is most easily seen by recognizing that each sub-function is itself a sum of different components. Since a linear sum of convex functions is itself convex, it suffices to show each component that constitutes the sub-function is convex. Consider the term $\alpha_{12}\ln(\mu/(p+\alpha_{122}w))\mu$. This can be separated into components $\alpha_{12}\ln(\mu)\mu$ and $-\alpha_{12}\ln(p+\alpha_{122}w)\mu$. The former is convex because the second derivative is $\alpha_{12}/\mu \geq 0$, for $\alpha_{12} \geq 0$ and $\mu > 0$. The latter is convex because the Hessian matrix of second derivatives is

$$\frac{\mu}{(p+\alpha_{122}w)^2}\begin{bmatrix} 1 & \alpha_{122} & \dfrac{p+\alpha_{122}w}{\mu} \\ \alpha_{122} & \alpha_{122}^{\ 2} & \dfrac{\alpha_{122}(p+\alpha_{122}w)}{\mu} \\ \dfrac{p+\alpha_{122}w}{\mu} & \dfrac{\alpha_{122}(p+\alpha_{122}w)}{\mu} & 0 \end{bmatrix}$$

with non-negative principal minors for $\mu > 0$. All elementary components to the profit function can be verified to be convex in this manner.

To illustrate the approximation properties of this model, consider now the structural equation for end wealth of a male head of household. The Frisch asset equation is found by differentiating the parent profit function with respect to r to give

$$-A^{*} = \partial\pi/\partial r = \pi_r^{\alpha}(P,\mu,\alpha)\mu + \pi_r^{\beta}(P,\beta:z) + \pi_r^{\gamma}(P,\gamma)/\mu \tag{10}$$

with differentiated sub-functions

$$\pi_r^{\alpha}(P,\mu,\alpha) = -\left(\frac{\alpha_{13}\alpha_{133}}{p+\alpha_{133}r} + \frac{\alpha_{23}\alpha_{233}}{w+\alpha_{233}r} + \frac{\alpha_{33}}{r}\right)$$

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[8] Additional terms with β parameters of the form $\beta_{ij}p_i^{0.5}p_j^{0.5}$ with convexity restriction $\beta_{ij} \leq 0$ were also tried

$\pi_r^{\beta}(\mathbf{P},\beta:z) = \beta_{130} + \beta_{131}\mathbf{age} + \beta_{132}\mathbf{age}^2 + \beta_{133}\mathbf{age}^3 + \beta_{134}\mathbf{dependents}$ and

$\pi_r^{\gamma}(\mathbf{P},\gamma) = \left(\left(\gamma_{11}p + \gamma_{12}w + \gamma_{13}r\right)\gamma_{13} + \left(\gamma_{22}w + \gamma_{23}r\right)\gamma_{23} + \gamma_{33}^2 r\right).$

A total of 16 parameters determine the wealth equation of male headed households. A total of 15 parameters determine the labor supply equation and 9 parameters determine the consumption equation.

Each structural equation such as (10) conditions multiplicatively on μ and $1/\mu$. Together with the derivative of the sub-function π^{β} which gives the third function, this Frisch system might be called rank 3 (Lewbel, 1991) drawing obvious analogies with indirect utility functions where μ replaces income. Additionally as μ and $1/\mu$ enters the structural equations, this can be considered as a first order Laurent approximation in μ. As demonstrated theoretically by Barnett (1983), the Laurent series approximation has superior fit compared to a Taylor series approximation of the same order and subsequently led to the Minflex family of demand systems.

This profit function has considerable generality which I highlight by drawing analogies to flexible functional forms. The unrestricted off-diagonal parameters γ_{12}, γ_{13} and γ_{23} identify the off-diagonal cross-price responses of matrix **N** in (4). The unrestricted diagonal γ_{22} and γ_{33} parameters identify how the structural equations change with respect to changes in $1/\mu$ and parallel the parameters that identify income responses in indirect utility systems.[9] The unrestricted β parameters identify levels with sufficient parameters for flexibility and additionally capture suspected demographic and lifecycle influences. The

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in the regression. These constraints were binding in every regression performed.

diagonal parameters α_{22} and α_{33} are analogous to the third rank in rank 3 systems[10] while the off-diagonal parameters α_{12}, α_{13} and α_{23} identify how the structural equations change with respect to cross-price terms conditional on μ for additional generality.

The economic restrictions of consumption-labor additivity and time separability are now easily cast as simple parametric restrictions on the structural equations which are described now in order of increasing generality. The first of these I call the basic regression, the most parsimonious case. This regression sets all cross terms $\alpha_{ij} = 0$ and $\gamma_{ij} = 0$ for $i \neq j$ and is implied by consumption-leisure additivity and time separability. This sets all off-diagonal elements of matrix **N** in equation (4) to zero. Additionally, all β parameters except 6 β_{ij0}, i=1, 2 and j=1, 2, 3, are set to zero removing the impact of age and number of dependents from the structural equations.

I illustrate the basic regression for a male head of household. The structural equations for real consumption, labor supply and assets are respectively,

$$\begin{aligned} -c &= -\alpha_{11}\mu/p + \beta_{110} + \gamma_{11}p\lambda \\ h &= -\alpha_{22}\mu/w + \beta_{120} + \gamma_{22}w\lambda \qquad (11) \\ -A &= -\alpha_{33}\mu/r + \beta_{130} + \gamma_{33}r\lambda \end{aligned}$$

or structural equations for consumption expenditure, labor earnings and wealth,

$$\begin{aligned} -pc &= -\alpha_{11}\mu + \beta_{110}p + \gamma_{11}p^2\lambda \\ wh &= -\alpha_{22}\mu + \beta_{120}w + \gamma_{22}w^2\lambda \qquad (11') \end{aligned}$$

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[9] Sometimes γ_{11} is estimated while at other times, it is held as a constant. When estimated, there is additional generality. This is discussed later.

[10] I do not estimate α_{11} for reasons discussed later in the results section.

$$-rA = -\alpha_{33}\mu + \beta_{130}r + \gamma_{33}r^2\lambda$$

The first generalization allows demographic variation to impact the levels c, h, and A. One expects labor supply and wealth demand to follow lifecycle patterns and this is accomplished by estimating an additional 16 β parameters associated with age, age squared and age cubed and the number of dependents. I call this case the demographic regression.

The second generalization allows for consumption-leisure non-additivity. This is accomplished by allowing parameters α_{12} and γ_{12} to take on non-zero values. This case allows for the identification of the element c_w (and by symmetry h_p) in matrix **N** in equation 4 and if non-zero implies that element L_{ch} in matrix **M** is non-zero. I call this case the time-separable regression. The third generalization allows for intertemporal non-separability. This is accomplished by allowing the remaining parameters α_{13}, α_{23}, γ_{13} and γ_{23} to take non-zero values. This of course fills the remaining elements of matrix **N** and, if non-zero, implies matrix **M** has non zero elements. I call this case the general regression.

I now discuss the determination of unobserved μ. Cooper, McLaren and Wong (2001) use a similar approach in that unobservable μ is implicitly determined by a budget identity for a static representative consumer problem while McLaren, Rossitter, and Powell (2000) implicitly determine unobserved utility in an expenditure function. Both papers invert the unobserved variable numerically. In contrast, this paper uses a function with the special property that μ can be solved with an explicit form.

If beginning and ending wealth, exogenous income, wages and interest rates are known, the unobserved μ is then implicitly defined by the budget constraint

$W + y + p\pi_p + w\pi_w + r\pi_r = 0$. Expressing this more fully by explicitly showing the differentiation of the each of the sub-functions to the profit function, one has

$$
\begin{array}{llllllll}
W + y + & & & & & & & \\
p\pi_p^{\alpha}\mu & + & p\pi_p^{\beta} & + & p\pi_p^{\gamma}/\mu & + & & \\
w\pi_w^{\alpha}\mu & + & w\pi_w^{\beta} & + & w\pi_w^{\gamma}/\mu & + & & \\
r\pi_r^{\alpha}\mu & + & r\pi_r^{\beta} & + & r\pi_r^{\gamma}/\mu & = & 0 &
\end{array}
\tag{12}
$$

Multiplying (12) by μ provides a quadratic formula in μ. Similarly, multiplying (12) by λ provides a quadratic formula in λ. To collect, column-wise, like terms in (12) and for notational convenience, let

$$\pi_{\Sigma}^{\alpha} = \sum_{i=1}^{3} p_i \pi_{p(i)}^{\alpha} = (-\alpha_{11} - \alpha_{12} - \alpha_{13} - \alpha_{22} - \alpha_{23} - \alpha_{33})$$

$$\pi_{\Sigma}^{\beta} = W + y + \sum_{i=1}^{3} p_i \pi_{p(i)}^{\beta} = W + y + \pi^{\beta}(P, \beta : z)$$

$$\pi_{\Sigma}^{\gamma} = \sum_{i=1}^{3} p_i \pi_{p(i)}^{\gamma} = 2\pi^{\gamma}(P, \gamma).$$

The equalities above for π_{Σ}^{α}, π_{Σ}^{β} and π_{Σ}^{γ} hold because of homogeneity in prices of degree 0, 1 and 2 respectively. The positive roots of quadratic equation (12) expressed in these alternative forms are, respectively,

$$\mu = \frac{-\pi_{\Sigma}^{\beta} - \sqrt{\pi_{\Sigma}^{\beta\,2} - 4\pi_{\Sigma}^{\alpha}\pi_{\Sigma}^{\gamma}}}{2\pi_{\Sigma}^{\alpha}} \tag{13}$$

and

$$\lambda = 1/\mu = \frac{-\pi_{\Sigma}^{\beta} + \sqrt{\pi_{\Sigma}^{\beta\,2} - 4\pi_{\Sigma}^{\alpha}\pi_{\Sigma}^{\gamma}}}{2\pi_{\Sigma}^{\gamma}}. \tag{14}$$

Quite serendipitously, restrictions sufficient for convexity, $\alpha_{ij} \geq 0, \alpha_{iij} \geq 0$, lead to $\pi_{\Sigma}^{\alpha} \leq 0$ and together with the quadratic form $\pi_{\Sigma}^{\gamma} \geq 0$ become sufficient for a globally positive discriminant and a real root. These expressions for μ and λ, which are now solely in terms of observable variables, are substituted into the structural equations such as (10).

The sub-functions π^{α}, π^{β} and π^{γ} have an economic interpretation that is noteworthy. Basically, the sub-function π^{α} captures asymptotically the preferences of the infinitely rich which I define as those with μ approaching infinity. The sub-function π^{γ} captures asymptotically the preferences of the infinitely poor which I define as those with λ approaching infinity. The sub-function π^{β} occupies a middle ground and acts as an intercept and pivot point around which conditional-on-μ and conditional-on-λ demands radiate.

Now consider the numerator of equation (13), $-\pi_{\Sigma}^{\beta}-\sqrt{\pi_{\Sigma}^{\beta\,2}-4\pi_{\Sigma}^{\alpha}\pi_{\Sigma}^{\gamma}}$. As $\pi_{\Sigma}^{\beta}\to\infty$, the term $4\pi_{\Sigma}^{\alpha}\pi_{\Sigma}^{\gamma}$ becomes inconsequential in the discriminant and

$-\pi_{\Sigma}^{\beta}-\sqrt{\pi_{\Sigma}^{\beta\,2}-4\pi_{\Sigma}^{\alpha}\pi_{\Sigma}^{\gamma}} \to \; -2\pi_{\Sigma}^{\beta}$. On the other hand, as $\pi_{\Sigma}^{\beta}\to-\infty$,

$-\pi_{\Sigma}^{\beta}-\sqrt{\pi_{\Sigma}^{\beta\,2}-4\pi_{\Sigma}^{\alpha}\pi_{\Sigma}^{\gamma}} \to \; 0$. The numerator of equation (14), $-\pi_{\Sigma}^{\beta}+\sqrt{\pi_{\Sigma}^{\beta\,2}-4\pi_{\Sigma}^{\alpha}\pi_{\Sigma}^{\gamma}}$ acts in the opposite manner approaching 0 as $\pi_{\Sigma}^{\beta}\to\infty$, and approaching $-2\pi_{\Sigma}^{\beta}$ as $\pi_{\Sigma}^{\beta}\to-\infty$.

Finally, consider the denominator of equation (13). This is equal to $2(\alpha_{11}+\alpha_{22}+\alpha_{33})$ in the basic regression. Substituting (13) into (11'), it is now obvious that as $\pi_{\Sigma}^{\beta}\to\infty$, marginal increases in wealth are apportioned to consumption expenditure, the absence of work earnings (because of the negative sign) and end-of-period wealth in the ratio $\alpha_{11}/(\alpha_{11}+\alpha_{22}+\alpha_{33})$, $\alpha_{22}/(\alpha_{11}+\alpha_{22}+\alpha_{33})$ and, $\alpha_{33}/(\alpha_{11}+\alpha_{22}+\alpha_{33})$ respectively. This apportionment is identical to that of the Cobb-Douglas utility discussed in the primer. A similar operation occurs with the sub-function π^{γ} which determines how marginal increases in wealth are apportioned to the three choice variables as $\pi_{\Sigma}^{\beta} \to -\infty$ and is determined by the γ parameters and by prices. This apportionment determined by the parameters of sub-function π^{α} I shall call the "α-effect," and the apportionment determined by the parameters of sub-function π^{γ} I shall call the "γ-effect."

In estimation, the set of β parameters act as intercept parameters identifying the central location of the distribution of consumption, labor or wealth. Since one expects demographic and life cycle factors to have an important influence on preferences, a total of $22\,\beta_{ijk}$ parameters are added to distinguish household heads by gender, age and number of dependents in the system of structural equations and their significance evaluated empirically. I shall call both the pivotal nature of intercept identification and the impact of demographics identified by parameters in sub-function π^{β} the "β-effect."

The structural equations are non-linear in μ which is an important attribute of this model. This feature of my model solved a particularly challenging problem encountered with what might be called linear-in-μ or linear-in-λ rank 2 models. Consider the case where π^{γ} is identically zero and the structural equations depend on the appropriately differentiated sub-functions of π^{α}. The structural equations are then linear in μ. I found that the β parameters again estimated the location of the data but derived values of μ took positive and negative values as the regression pivots linearly around the central location of the distribution. Negative values for the price of marginal utility violate regularity and do not make economic sense. Negative values were found in approximately half the sample irrespective of whether the sub-functions π^{α} or π^{γ} were identically zero. The approach adopted here where the structural equations conditions on both μ and λ simultaneously solved this problem. As can be seen from explicit equations (13) and (14), solved values for μ or λ will always be positively valued.

As mentioned at the start of this section, beginning and end wealth is not available for just one year as has been supposed here- but fortunately, this is not a major complication. I turn next to a description of the data and how this was made operational despite data shortcomings.

DATA

The data for this study come from the Panel Study of Income Dynamics (PSID), a continuing study started in 1968 with approximately 4,800 households. Since that time, the panel has grown as the number of new households formed has exceeded those that have attrited (Hill, 1992). Five waves of the family files from surveys fielded from 1985 to 1989 were used which each contained detailed income and work data for the previous year. Additionally, these files include two wealth supplements that asked about wealth in the beginning and end of this span. By the time of interview, many households have had long standing participation with this survey.

To build a balanced panel across 5 years, single headed households who remained heads from 1984 to 1989 were selected. This choice does not allow for changes in what the PSID calls major adults but allows other changes to the households such as the birth or adoption of children or children leaving home to establish one of their own.

I removed all cases where income, wage or wealth was top-coded in any year by the PSID.

Next, a measure of real return on wealth for each year is calculated. I treat wealth as a single amorphous good (as the literature commonly does for consumption and labor supply) and add all disparate returns to wealth into a single total. I derive real returns by adding all recorded receipts from wealth: net of tax income received from financial assets such as rent, dividends and interests; and the asset portion of income from unincorporated businesses, farming, market gardening and roomers and boarders as determined by the PSID and net out the impact of marginal tax rates on this income. To this net of tax total, one fifth of the 5-year capital gain as determined by the PSID is added without tax on the assumption that the capital gain is unrealized and therefore not taxable. Finally, I add an imputed rental services on owner occupied housing. I treat the decision to purchase a home as an investment, rather than consumption, decision and therefore add its tax-free benefit to a general total to compute the return on wealth.

This total return on wealth was divided by an interpolated level of wealth based on net wealth in 1984 and in 1989. I use the variable net wealth as defined by the PSID which includes the main home, other real estate, farms or businesses, stocks, cash accounts and other items, but exclude the value of motor vehicles.

The income return on wealth, i_t, is given by

$$i_t = \frac{(1 - mt_t)\text{total asset income}_t + 0.05\text{house value}_t + \text{capital gain}/5}{(W_t + W_{t+1})/2} \tag{15}$$

with $W_t = W_{1984} + (t\text{-}1984)(W_{1989}\text{-}W_{1984})/5$ and mt_t equal to time-t marginal federal income tax rate.

In the money demand literature, for example Barnett, Fisher and Serletis (1992), expressions are sought for the return on different kinds of money assets and a household's return is based on their mix of these assets. This assumption is appropriate if the monetary asset classes are homogeneous and there is a competitive market which brings about one price (or return) for each asset class.

Here, I use directly the household's income and capital gain on assets to derive an individual specific return on assets. Cross-sectional variation in wages in the labor supply literature is often explained by differential productivities of workers such that firms are paying a fixed wage rate per unit of productivity. In a parallel fashion, I consider it reasonable that households may also have different productivities in obtaining yields on their assets. In any case, the return on wealth I consider- all disparate items that make up wealth- is much more heterogeneous than studies based narrowly on monetary assets. For instance, a proprietor of a business is likely to have intimate knowledge of the return on a particular investment in their business which will not be arbitraged with other proprietors of other businesses because of the absence of markets. My analysis is interested in the range of returns felt by households and their response to their idiosyncratic return rather than their response to returns available generally in markets.

All households with starting or ending wealth of less than \$500 were deleted. Some of these cases show implausibly large values for i_t which is understandable given the denominator of (15) is small. Additionally, all households with i_t greater than 40% or less than -20% in any of the five years were also deleted.

Next, a value for 5 year consumption is calculated for each household. This is calculated residually from the ex-post budget constraint. The budget constraint for a one year period is $W_t + y_t + w_t h_t - p_t c_t - r_t A_{t+1} = 0$. By recursive substitution, the 5 year budget constraint is

$$W_{1984} + \sum_{t=1984}^{1988} rr_t (y_t + w_t h_t - p_t c_t) - rr_{1988} r_{1988} A_{1989} = 0 \qquad (16)$$

where rr_{1984}=1 and $rr_j = \prod_{t=1985}^{j} r_t / p_{t+1}$, j=1985,...1988. While it is not possible to calculate consumption in each period, I can calculate 5 year composite consumption by

$$\sum_{t=1984}^{1988} rr_t p_t c_t = W_{1984} + \sum_{t=1984}^{1988} rr_t (y_t + w_t h_t) - rr_{1988} r_{1988} A_{1989} \qquad (17)$$

where all terms on the RHS are observable and based on ex post prices.

Using this measure of consumption, several cases of negative consumption expenditures were observed. For these cases, final assets were too high given beginning assets and recorded incomes. These cases were deleted as were cases which had calculated 5 year consumption expenditures less than $5,000 which I arbitrary define as a subsistence level of expenditure.

The term y_t which measures time t exogenous income was calculated as income from all public and private transfers plus inheritances plus federal marginal tax rates times pretax labor earnings less federal income taxes for year t. Since I wish to capture household decisions at the margins, it is appropriate to use $(1\text{-}mt_t)$pretax $wage_t$ as the real benefit of working the marginal hour in year t. However, since we have a progressive income tax system, marginal tax rates multiplied by gross labor earnings overstates the amount of tax paid on labor earnings. As the objective of the consumption equation (17) is to calculate the present value of consumption from observables, I add back marginal tax multiplied by earning and subtract federal income taxes since this information is available. This has the effect of adding what many economists call *virtual income*.

After removing observations as described, 525 observations were left. Summary statistics for both male and female headed households are recorded in the table 1.

Table 1. Summary statistics of selected sample in 1988 by gender.

	Male headed households (n=92)				Female headed households (n=433)			
Variable	mean	std. dev.	minimum	maximum	mean	std. dev.	minimum	maximum
Age of Head	56.5	17.6	27	90	63.1	16.2	25	97
# of dependents	1.3	0.75	1	6	1.3	0.81	0	8
Consumption ($)	80,178	45,055	10,802	209,461	61,130	39,823	6,397	270,458
Wealth ($)	73,950	79,257	1,500	439,000	69,587	91,392	1,200	825,000
Interest return (i)	0.071	0.086	-0.184	0.335	0.076	0.093	-0.199	0.338
Exogenous income ($y)	1,998	8,643	-21,918	34,320	5,172	15,116	-10,399	255,953
Head work hours *	1,907	586	120	2,880	1,666	611	10	2,975
Labor Earnings ($) *	16,144	13,243	441	66,861	10,243	8,120	65	42,947

* summary statistics reported only for the 61 males and 203 females working in 1989.

The model counterpart to the ex-post 5-year budget constraint (16) is accomplished by supplying the corresponding Frisch demand for each quantity. Naturally, I will supply the prices that prevailed at the time the choice was made. In each period, household exercised 3 choices: a consumption, work and savings decision, maximizing their value function (1). I wish to impose the minimum structure necessary to estimate parameters of the model. So, although households made 5 endogenous A_t choices for t= 1985,…, 1989, since only the final year is available in the data, I do not impose any structure on the earlier unknown wealth choices. Similarly, the household made 5 endogenous c_t choices for t= 1985,…,1989 which cannot be determined individually. I impose no structure on the 5 year ex-post budget (16), other than the fact that the appropriate time t Frisch demand represents the choices made. Thus, the model counterpart to the budget constraint (16) is

$$W_{1984} + \sum_{t=1984}^{1988} rr_t (y_t + w_t \pi_w(t) + p_t \pi_p(t)) + rr_{1988} r_{1988} \pi_r(1988) = 0 \tag{18}$$

It can be seen from profit function (5), which in turn is derived from value function (1), that Frisch demands represent a household's best effort at intertemporal optimization given time t information. It is certainly possible for a household to regret a decision made previously given the realization of new information. It is also true that a household will evaluate future prices as it affects the expected next period value function in making their present choices but that this choice is made in light of present information only.

The model counterpart of the budget constraint (18) reveals another advantage of the dual approach I have used. In the dynamic context, consumption and work are control variables while asset is a state variable. Ending asset is an endogenous variable if this choice is modeled; however it becomes a predetermined, weakly exogenous variable in the following period. When a primal approach is used for the specification of preferences, one must employ recursive substitution schemes but this becomes intractable when there is any generality to the utility function. The dual approach I employ adds an additional state variable, λ, which I can model independently.

My approach gives rise to 7 structural equations that can be matched with the data: one for 1989 wealth, one for 5 year composite consumption and 5 for labor supply in each of the years 1984 to1988. However, consumption was not independently determined but rather imputed from (17), a function of 1989 wealth and 5 year's labor supply. Thus, I use the 6 independent estimating equations;[11]

$$w_t h_t = w_t \pi_w(t) + \varepsilon_t \qquad \text{for } t=1984,\ldots 1988, \text{ and}$$

$$r_{1989} A_{1989} = -r_{1989} \pi_r(1988) + \varepsilon_{1989} \tag{19}$$

appending ε_t as period t error. I assume that $\varepsilon = (\varepsilon_{1984}, \varepsilon_{1985}, \varepsilon_{1986}, \varepsilon_{1987}, \varepsilon_{1988}, \varepsilon_{1989})'$ is multivariate normal and estimated parameters using the full information maximum likelihood procedure implemented in TSP version 4.5.[12]

I multiply the wealth equation by r_{1988} and each period t labor supply equation by w_t, a practice common in the demand analysis field that allows for adding up in the budget constraint. For my purpose, it also has the effect of removing households with non-working heads from the regression.[13] Thus I use all the 525 observations available to estimate the parameters while only those working in any year contribute to the estimation of parameters

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[11] A consumption expenditure estimating equation could be added to the list but the error from this equation is not independent of the other 6 errors as the sum of all 7 errors is identically zero.
[12] The regression output from the TSP estimation is shown in the appendix.

associated with wages and labor supply in the year they worked. While this is only a subset of the sample (see table 1), I have 5 years of repeated measures recording the head's work choices.

As the 6 estimating equations (19) stand, they contain 5 unobserved arguments, μ_t while the single 5 year budget constraint (18) allows me to solve only one additional variable. My strategy is to use the budget constraint to determine an individual specific μ_i and consider a menu of 3 choices which relate μ_{it} to μ_i where I introduce the i indexing subscript to denote household i. These choices will be guided by what readers feel are reasonable behavioral assumptions about the ability of households to intertemporally optimize. I am not partisan to any specific formulation on the evolution of μ and consider it simply an empirical matter.

Each of these alternatives will have the form $\mu_{it} = f(p, w_i, r_i, W_i : \delta, t)\mu_i$ where p, w, r, and W denote a 5 year vector containing the corresponding price, wage, interest factor or wealth series, δ denotes a time discount factor and the subscript denotes household i, and can be substituted into equation (18). Function f is required to be homogeneous of degree zero in prices and nominal wealth. Further, function f in its most general form exhausts all price and wealth information available in my analysis. The implicit definition of μ in equation (18) makes this, like function f, a function of nearly all available information in the general regression. I show, however, that these can be made to play different roles.

The first of my models supplying the 5 required functions I call the fixed effects model and corresponds to $\mu_{i,t} = (p_t / p_{1988})\mu_i$. The fixed effects model holds the real price of

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[13] My purpose is not to explain the dichotomous decision to work or not for which a reservation wage needs to

marginal utility constant for each household i for the 5 time periods t. Cross-sectional differences in beginning wealth, exogenous income, wages and interest rates are the sole factor creating cross-sectional differences in the real price of marginal utility in this specification. The second case I offer I call the perfect foresight model and is defined by $\mu_{i,1988} = \mu_i$ and $\mu_{i,t} = p_t / p_{1988} \prod_{j=t}^{1987} \delta r_{i,j} / p_j \mu_i$. This specification corresponds to a perfect foresight Euler equation with time discount parameter, δ, an additional parameter requiring estimation. As in the fixed effects model, cross-variation drive differences in the price of utility for the year 1988, however, household specific time t real price of utility for other years are driven by household interest returns. The third case I offer I call the stochastic model where $\mu_{i,t}$ is perturbed around μ_i by realizations of household specific real time t variables. The stochastic model specifies

$$\mu_{i,t} = \delta^{(1988-t)} p_t / p_{1988} \exp(\theta_w (\tilde{w}_{i,t} - \tilde{w}_i) + \theta_r (\tilde{r}_{i,t} - \tilde{r}_i) + \theta_a (A_{i,t} - A_i)) \mu_i \qquad (20)$$

where δ, θ_w, θ_r and θ_a are 4 additional parameters requiring estimation and tilde $\tilde{}$ denotes the real wage, $w_{i,t}/p_t$, or real interest factor, $r_{i,t}/p_t$. The term $w_i = \sum_{t=1984}^{1988} w_{i,t} / 5$ is household i's average of real wage over the 5 years observed. It can be interpreted as an analyst's estimate of a household's "permanent wage" similar in concept to permanent income used in intertemporal models. Household real interest return and assets are defined and interpreted similarly. This specification allows for time discounting of real marginal utility. The exponential component incorporates shift parameters θ when real wages, real interest factors or assets are perturbed in time t from a household's 5 year average.

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be constructed.

RESULTS

This section reports on some of the challenges of estimation, how these were overcome and results obtained. As is common in non-linear estimation, functions of parameters were often easier to identify that the parameters individually. I found that the remaining α_{ij} and γ_{ij} parameters were far easier to estimate once one non-zero α and one non-zero γ were specified as constants. This is understandable if one considers the structural equations. Consider the (negative) wealth expenditure equation below,

$$-rA^{*} = r\partial\pi/\partial r = r\pi_r^{\alpha}(P,\mu,\alpha)\mu + r\pi_r^{\beta}(P,\beta : z) + r\pi_r^{\gamma}(P,\gamma)/\mu$$

and the explicit function for μ,

$$\mu = \frac{-\pi_{\Sigma}^{\beta} - \sqrt{\pi_{\Sigma}^{\beta\,2} - 4\pi_{\Sigma}^{\alpha}\pi_{\Sigma}^{\gamma}}}{2\pi_{\Sigma}^{\alpha}} .$$

It can be seen on substitution that the wealth equation changes with the term $-\pi_{\Sigma}^{\beta} - \sqrt{\pi_{\Sigma}^{\beta\,2} - 4\pi_{\Sigma}^{\alpha}\pi_{\Sigma}^{\gamma}}$ in the proportion $r\pi_r^{\alpha}/2\pi_{\Sigma}^{\alpha}$ which is simply a ratio of α parameters. The same is also true of the γ parameters where the wealth equation changes with the term $-\pi_{\Sigma}^{\beta} + \sqrt{\pi_{\Sigma}^{\beta\,2} - 4\pi_{\Sigma}^{\alpha}\pi_{\Sigma}^{\gamma}}$ in the proportion $r\pi_r^{\gamma}/2\pi_{\Sigma}^{\gamma}$. Except for the appearance of the α and γ parameters in the discriminant, the structural equations would be homogeneous of degree zero in these parameters. For this reason, it is not possible to estimate all α and γ parameters simultaneously. I arbitrarily set $\alpha_{33}=1$, an innocuous specification because multiplying the α_{ij} parameters and dividing the γ_{ij} parameters by any positive constant leaves the structural equations unchanged.

With the parameter α_{33} set to one, attempts were made to estimate the other parameters however this was sometimes not successful. In some of the restricted regressions, the γ parameters could be estimated however for some of the more general specifications, it was found that the set of γ parameter would uniformly converge to zero. This behavior increased the number of iterations and squeeze steps required before the TSP program was terminated either successfully by meeting specified parameter convergence criteria, but most often unsuccessfully because it exceeded specified iteration or squeeze step limits within the TSP procedure.

Consider now the functional forms of μ and λ as the parameters γ_{ij} converge to zero. In this case, μ converges to $\min[\pi^{\beta}_{\Sigma},0]/\pi^{\alpha}_{\Sigma}$ and λ converges to $\max[\pi^{\beta}_{\Sigma},0]/\pi^{\gamma}_{\Sigma}$ as the discriminant converges to ${\pi^{\beta}_{\Sigma}}^2$. This creates linear segments to the structural equations in μ and λ, pivoting around the β parameters. Because it is arguably more appealing that the structural equations should be "smooth" in μ and λ, because the likelihood changed little as the γ parameters converged to zero and, in particular, because of the difficulties of obtaining successful convergence as the γ_{ij}'s approached zero, γ_{11} was sometimes set as a constant.

Other specifications that aided estimation were holding parameters γ_{11}, γ_{22} and γ_{33} to be non-negative. It can be seen from the structural equations that these parameters are squared. Consequently, it is innocuous for the fit of the regression to use the positive values of these diagonal γ_{ii} parameters or truncate it at zero if the likelihood function is decreasing in this parameter.[14] Without this specification, the parameter γ_{ii} often seemed to oscillate explosively around zero for cases where likelihood was maximized at γ_{ii}=0. This truncation was also applied to the α_{ij} parameters to ensure non-negativity. Despite these aids to estimation, I found that in no case was I able to estimate parameters α_{ijj} so these were arbitrarily left at one. Thus, in the results I report below, some of the parameters are unrestricted while others are bound by the convexity restrictions or held as constants. Furthermore, the set of parameters that were binding differed depending on the regression performed. Nonetheless, the parameter estimates I report here are convergent and robust in the sense of being independent of the many starting values I tried. I describe these regressions next.

For each of the three models: fixed effect, perfect foresight, and stochastic models; I performed four regressions: the basic, demographic, time-separable and general regressions. The log likelihoods of these 12 regressions and the number of free and constrained parameters are summarized by the 4x3 cells of table 2.

In each of the 12 cells of table 2, the top row gives the log likelihood of the corresponding model and regression. The second row of each cell gives two numbers: the total number of parameters estimated *within* the parenthesis and the number of free parameters estimated *outside* the parenthesis. The difference between the numbers inside and outside the parenthesis is the number of α_{ij} and γ_{ij} parameters constrained at zero due to convexity restrictions or held as a constant. The perfect foresight model involves one additional parameter over the fixed effect model, the time discount parameter δ. The

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[14] This was specified in TSP as $\gamma_{ii} = (\varphi_{ii} > 0)\varphi_{ii}$ where φ_{ii} was the parameter actually submitted into the

stochastic model involves 4 additional parameters over the fixed effects model, the parameters δ, θ_a, θ_r, and θ_w. While the perfect foresight model is not nested in the fixed effect model, the likelihood result suggests that it is a somewhat better fit than the fixed effects model. The estimate for the time discount factor parameter ranged from 0.61 to 0.91 in the four regressions with a value of 0.84 for the general regression.

Table 2. Log likelihood fit of various regressions and models

	Fixed Effect Model	Perfect Foresight Model	Stochastic Model
Basic Regression	-29,982.8 9(10)	-29,947.9 10(11)	-29,610.7 15(15)
Demographic Regression	-29,550.6 24(26)	-29,543.8 26(27)	-29,363.6 31(31)
Time Separable Regression	-29,539.2 25(28)	-29,533.5 26(29)	-29,354.6 32(33)
General Regression	-29,533.0 26(32)	-29,528.4 26(33)	-29,299.0 30(37)

The fixed effect model is nested in the stochastic model with θ_a, θ_r and θ_w all set to zero and δ set to one. It is clear on the basis of the likelihood ratio test that the hypothesis that these parameters are equal to zero is easily rejected in every regression. For this reason, and because the stochastic model allows for the determination of long-term and intertemporal effects I discuss later, I focus remaining discussion on the demographic, time separable and general regression of the stochastic model.

Table 3 reports parameter estimates for the demographic, time separable and general regressions for the stochastic model. Blank cells represent parameters not part of the model while cells with a value marked by an asterisk denote a parameter held at a constant, either because of a convexity restriction or to aid estimation as described above. The only case of the latter is in the stochastic model is parameter γ_{11} which was held in the general regression

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regression. Within the parenthesis is a logical function that takes the value 1 if true and 0 if false.

at the same value it was in the time-separable regression. I found that the likelihood was exceedingly flat in this parameter but γ_{11} tended to drift towards zero, together with other γ parameters without a convergent solution.

Recall that I shall call the direct impact of sub-functions π^{α}, π^{β} and π^{γ}, the "α-effect," the "β-effect" and the "γ-effect" respectively[15]. As discussed above, this captures the preferences of the infinitely rich, those in the middle and the infinitely poor.

One can see from parameter α_{11} in the demographic and time-separable regressions that the marginal propensity to spend additional wealth on consumption is 3% for the infinitely rich. The marginal propensity to reduce labor earnings captured purely by parameter α_{22} in the demographic regression and captured as a cross price effect by parameter α_{12} in the time-separable regression both suggest that this effect is economically small, around 0.5%, although statistically significant. In contrast, in the general regression, all estimated α parameters were bound by the convexity constraint suggesting that the propensity to consume or reduce labor earnings is zero for the infinitely rich.

The above α effect can be understood in the light of changes to the β parameters across the regressions. The parameters capturing the intercepts and pivot points of consumption, labor and wealth are, respectively, β_{110}, β_{120}, and β_{130} for male headed households. The corresponding parameters for female headed households are β_{210}, β_{220}, and β_{230}, respectively. In both the demographic and time-separable regressions, the β effect centers consumption at around \$24,000 for males and \$27,000 for females. In the general regression, the consumption pivot point shifts substantially, to \$100,000 for males and \$140,000 for females.

The change in the pivot points across regressions also occurs for labor and wealth. The demographic and time-separable regressions find an intercept of 1,860 and 1,400 hours per annum for male and female labor supply respectively while in the general regression the corresponding figures are 100 and -1,300 hours per annum respectively. It seems that there is also a change in the pivot points for wealth across the regressions however this is measured with considerably less precision than consumption or labor. The standard errors on the wealth intercepts are around an order of magnitude greater than the standard errors on consumption which are both measured in dollars. Nonetheless, it seems wealth pivots around \$0 for males and \$60,000 for females in the demographic and time-separable regression which shifts substantially to \$900,000 and \$1,400,000 for males and females respectively in the general regression.

The demographic β parameters which allow the structural equations to conditions on age, age squared, age cubed and number of dependents show consistency across the applicable regressions. The age variable I use is reported age of head minus 45 years in order to center the regression around prime aged heads. A comparison of parameters β_{120} and β_{220} show that- conditional on marginal utility, wages and interest rates- males at 45 work longer hours than females of the same age. However, parameters β_{121} and β_{221} show that males work 35 hours less per annum in the following year compared to females who work 20 hours

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[15] The parameters have a direct and linear impact on the structural equations conditional on μ, however by a circuitous route though μ, may have further effects. The α, β and γ effects are used to mean the conditional impact.

Table 3. Parameter estimates of the stochastic model across 3 regressions

	Demographic		Time Separable		General	
Parameter	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
α_{11}	0.03318	0.0059283	0.032804	0.0066956	0*	
α_{12}			0.0056241	0.0006993	0*	
α_{13}					0*	
α_{22}	0.0046038	0.0005638	0*		0*	
α_{23}					0*	
β_{110}	-23375.8	1209.77	-24125.9	1467.96	-99979.3	20191.4
β_{210}	-25630.4	1125.53	-27538.2	1416.5	-140612	29660.6
β_{120}	1859.11	34.4243	1856.47	36.237	100.242	467.995
β_{121}	-34.7792	1.9906	-33.7884	1.97332	-35.6121	1.99509
β_{122}	-1.44044	0.136691	-1.47057	0.137078	-1.91243	0.160302
β_{123}	0.021471	0.0054478	0.020706	0.005256	0.033636	0.0054422
β_{124}	-3.63477	12.9627	-0.858932	12.0349	11.0151	11.5607
β_{220}	1407.97	30.629	1385.28	32.9326	-1329.3	695.593
β_{221}	-20.3276	2.06151	-20.4163	1.98797	-17.1739	1.94799
β_{222}	-0.855218	0.123273	-0.823363	0.121752	-0.605163	0.121535
β_{223}	0.012911	0.0055697	0.010088	0.0053887	0.0030431	0.0052586
β_{224}	39.5843	15.3562	38.6231	15.4754	32.456	15.6588
β_{130}	-1165.4	17670.4	7239.08	18687.1	-877141	224388
β_{131}	-4522.81	776.534	-4351.38	852.572	-3989.56	837.085
β_{132}	-162.618	53.8203	-192.06	56.727	-367.03	56.7942
β_{133}	4.62172	1.55768	5.08941	1.65942	8.64496	1.57291
β_{134}	15037.4	18890.5	15810.5	15630.6	24616.1	15050.4
β_{230}	-54748.2	6840.44	-59631.4	8248.83	-1412420	329417
β_{231}	-2484.33	444.885	-2741.18	482.715	-3489.06	710.547
β_{232}	-117.697	30.8212	-115.153	32.4794	-98.3309	44.9346
β_{233}	2.90471	0.689035	2.9062	0.720474	2.95763	0.923903
β_{234}	-6645.33	4575.37	-5360.08	4905.41	-1470.74	7236
γ_{11}	9637.1	1135.53	13006.1	1783.5	13006.1*	
γ_{12}			245.936	67.5396	14.8286	60.2521
γ_{13}					76143.1	17853.1
γ_{22}	756.763	82.8449	722.453	84.1559	350.532	54.6282
γ_{23}					77353.3	3645.78
γ_{33}	21946.8	2215.93	29773.1	3634.84	0*	

0* indicates a binding convexity restriction on the parameter.

Table 3 (continued). Parameter estimates of the stochastic model across 3 regressions

	Demographic		Time Separable		General	
Parameter	Estimate	Std. Error	Estimate	Parameter	Estimate	Std. Error
θ_w	0.229225	0.010115	0.195287	0.011549	0.020112	0.0043826
θ_r	0.348836	0.479909	0.465873	0.4207	1.01533	0.058141
θ_a	1.06E-05	1.731E-06	1.02E-05	1.489E-06	1.59E-06	4.011E-07
δ	0.9334	0.01467	0.92617	0.01421	0.99022	0.00324

per annum less the following year. The parameters β_{122} and β_{222} show that the decline in work hours past 45 years of ages is occurring at an increasing rate for both males and females. A comparison of parameters β_{130} and β_{230} show that females at 45 desire more wealth than males of the same age. Parameters β_{131} and β_{231} show that males desire an increase of \$4,000-\$4,500 in wealth in the following year after age 45 compared to females who desire an increase of \$2,500-\$3,500. The parameters β_{132} and β_{232} show that the desire to increase wealth is increasing past 45 years of ages for both males and females.

Among the parameters that capture the effects of dependents on labor and wealth for males and females, only one, β_{224}, is statistically significant. This estimate suggests females work around 38 hours more per annum for each dependent she has. Although not statistically significant, the estimates are suggestive that males and females differ in their savings in response to the number of dependents. Each dependent reduces the wealth of males by around \$15,000-\$25,000 while it seems females save or provide an additional \$1,500-\$6,600 for each dependent. These results must be tempered in the light of the standard errors associated with the estimate.

The evaluation of the γ effect is comparatively more difficult than it is for the α and β effects. The reason for this is that the γ parameters multiply prices. The structural equations of the basic regression (13) illustrate this. For illustrative purposes, let p = 1, w = 4.1 and r = 0.93. For the demographic regression using parameters $\gamma_{11}, \gamma_{22}, \gamma_{33}$, a one unit increase in λ decreases consumption by \$9,600 and wealth by \$4,870 and increases work hours by 725 hours annually.

Table 4 gives quantitative indicators of fit for the system of 6 equations for the demographic, time separable and general regressions for the stochastic model. The R-square statistic are for the labor earnings equation $w_t h_t = w_t \pi_w(t) + \varepsilon_t$ equation for t = 1984 to 1988 and wealth equation $r_{1988} W_{1988} = r_{1988} \pi_r(1988) + \varepsilon_{1988}$. Looking across the regressions, relaxing consumption-labor additivity improves the fit of the labor supply equations although the fit of the wealth equation falls. Despite this, it can be seen from the likelihoods and from table 3 that the two additional parameters, α_{12} and γ_{12}, are significant. By relaxing time separability, there is a comparatively large increase in the likelihood which is mainly attributable to the improved fit of the wealth equation.

Table 4. Statistics of fit for labor earnings and wealth equations for stochastic model across demographic, time separable and general regressions.

Statistic of Fit	Demographic	Time Separable	General
R^2: labor 1984	0.880	0.881	0.885
R^2: labor 1985	0.891	0.892	0.893
R^2: labor 1986	0.917	0.918	0.918
R^2: labor 1987	0.924	0.926	0.930
R^2: labor 1988	0.931	0.933	0.933
R^2: wealth 1989	0.799	0.797	0.827
Log. Likelihood	-29,363.6	-29,354.6	-29,299.0

I show the qualitative fit of the consumption, labor and wealth equations for the general regression in figures 1.a, 1.b and 1.c respectively. In these figures, I plot actual values on predicted values. If my model predicted perfectly, all points would lie on a 45 degree line. It seems that the model fits fairly well without obvious systematic errors. There appears to be some heteroskedasticity for the consumption equation although this doesn't appear as strong in the labor earnings or wealth equation. Note however that the consumption equation was not part of the system of estimating equations. One can also see the truncation of actual wealth I used. My wealth equation gave negative predictions for a small portion of the sample.

My final analysis of this model looks at the impact of restrictions that are commonly employed in the literature and their impact on various estimated elasticities. For the structural equation using say end of period assets as an example, $-A^* = \pi_r(p, w, r, f\mu)$, I define Frisch elasticity as the derivative of the log of asset with respect to the log of any of the arguments and denote this ε^F_{Aj}, j={p,w,r, μ }. The first item in the subscript denotes the choice variable and is c for consumption and h for labor hours. I define short term Marshallian elasticities by recognizing the dependence of μ on prices and beginning wealth. Again using end of period assets as an example, $-A^* = \pi_r(p, w, r, f\mu(p, w, r, W))$, I define short term Marshallian elasticity of end of period asset with respect to j, $\varepsilon^M_{Aj} = \varepsilon^F_{Aj} + \varepsilon^F_{A\mu}\varepsilon^\mu_{\mu j}$ where ε_{Aj} denotes the elasticity with respect to j={p,w,r,W} and superscript M and F denote Marshallian and Frisch elasticities respectively and superscript μ denotes the elasticity of μ with respect to any of its arguments. It should be understood that the Frisch elasticity with respect to beginning wealth is zero since Frisch elasticities condition on μ, not wealth.

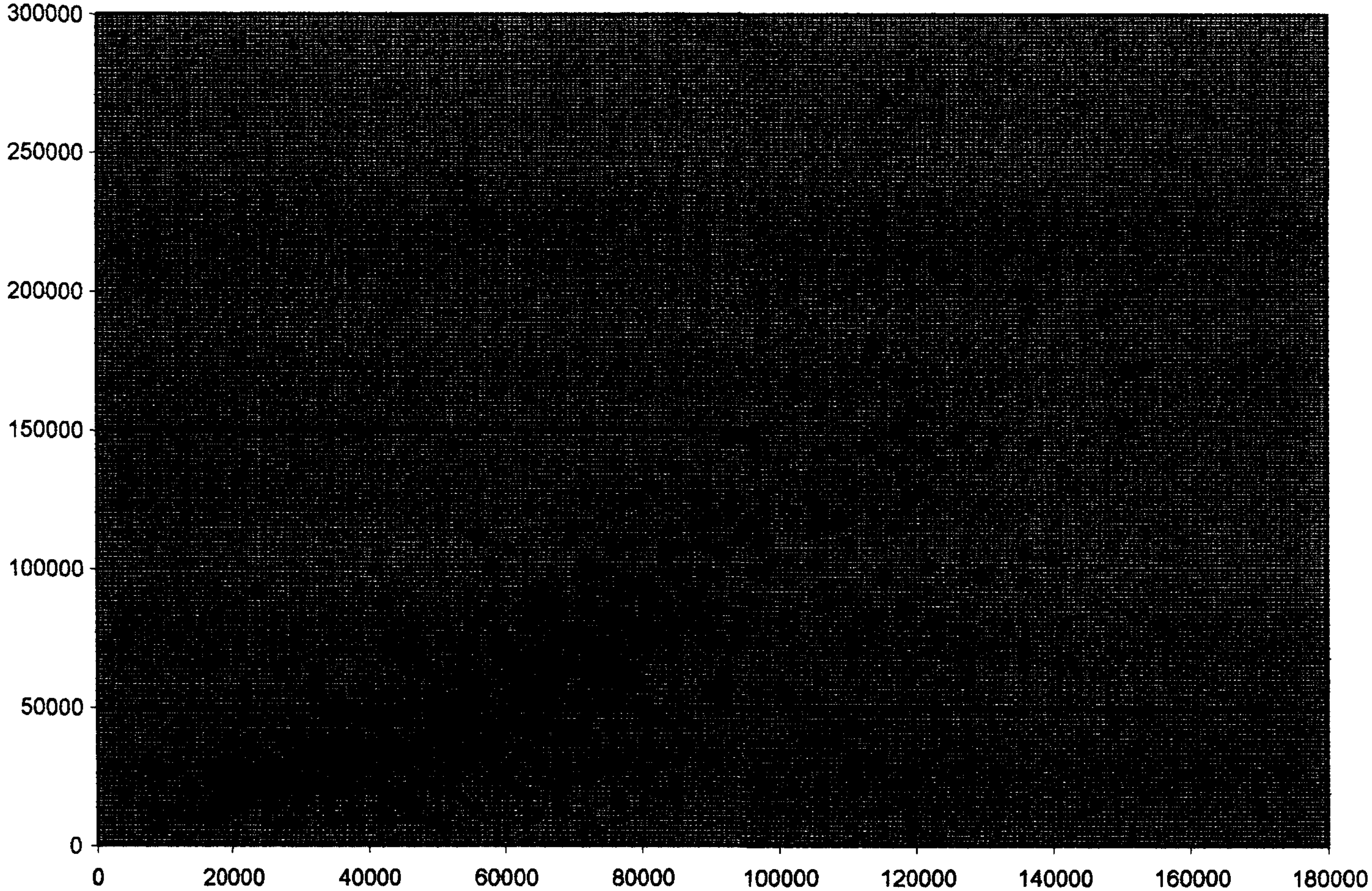

Figure 1.a. Actual 5 year consumption expenditure on predicted consumption in dollars

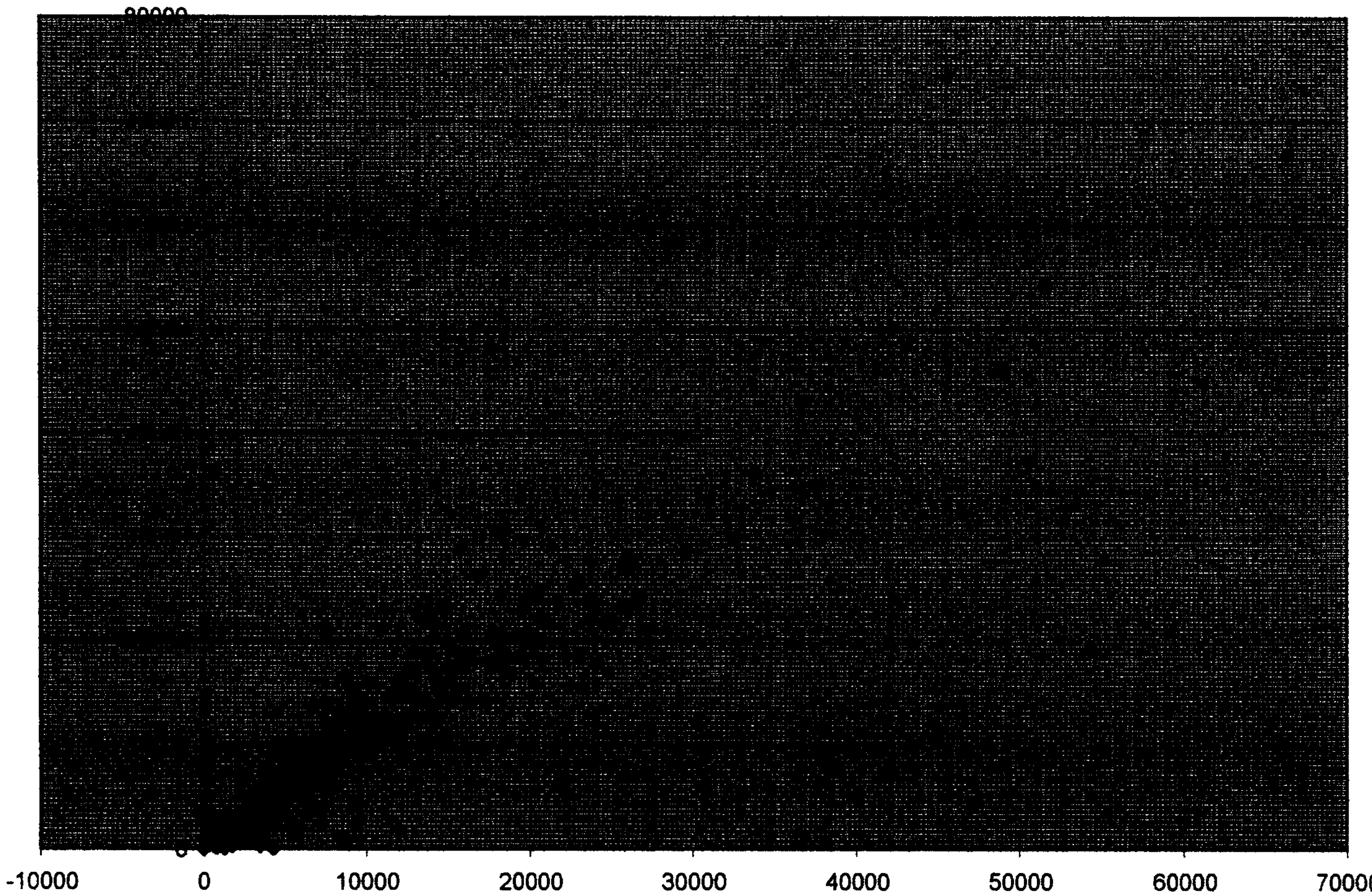

Figure 1.b. Actual labor earnings on predicted earnings in dollars

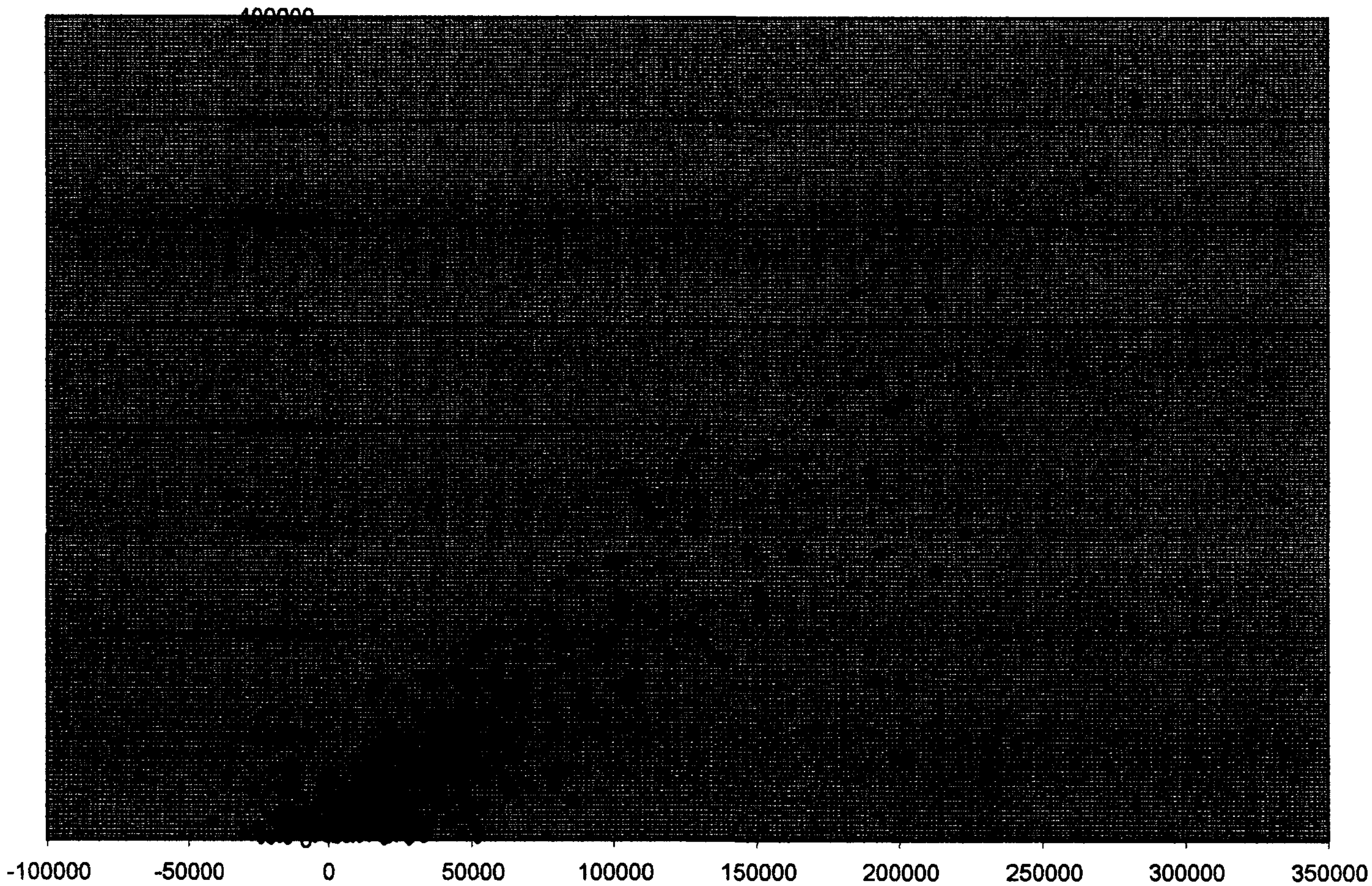

Figure 1.c. Actual wealth on predicted wealth in dollars.

I report Frisch and Marshallian elasticities for rich which I define as those with beginning wealth and exogenous income, W = 100,000 and poor households which I define as those with W = 50,000. Other prices, held at sample means, are p = 1, w = 4.1 and r = 0.93 and I assume the individual is 45 years of age with no dependents. Table 5 reports predicted consumption, labor and wealth of rich and poor householders which should be read in conjunction with table 1 which reports mean values of the sample for interpretation.[16]

Table 5. Predicted consumption, labor and wealth for rich and poor households.

	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
Assets (1989 dollars)	88377.9	877.854	88063.8	941.275	86998.3	1151.07
Consumption (1989 dollars)	25147.2	810.475	25512.8	867.72	26890.4	1060.93
Annual Work Hours	1789.92	25.9852	1807.82	25.8248	1902.15	29.1025
	Poor Household					
Assets (1989 dollars)	37594.3	979.128	37680	1077.12	38358.8	1164.91
Consumption (1989 dollars)	22681.1	890.06	22730.1	967.397	22562.6	1073.54
Annual Work Hours	1864.33	27.7772	1895.72	30.9583	2008.85	29.2107

Table 5 can be evaluated in many respects. In comparing between rich and poor households it seems that the main impact of greater wealth is the perpetuation of high wealth holdings. Rich households consume more than poor households although the increment is proportionately less than it is for wealth. Note also that the rich work less than the poor by about 80-100 hours annually. Additionally, although it seems that there were substantial changes in the α, β and γ effects discussed above between the restricted regressions and the

[16] The consumption figure in table 1 is actual 5 year composite consumption (see equation (16)) whereas the consumption figure here is predicted one year consumption. Otherwise, actual figures of table 1 and the predicted figures of table 5 are comparable. Additionally, I am not attempting to predict the means of table 1 but rather to give readers a sense of where the hypothetical rich and poor households are situated with respect to the actual data.

most general regression, these changes do not manifest themselves to a great extent in predicted quantities. End of period assets and consumption are comparable across the regressions while the predicted hours worked is around 100 hours more for the general regression than it is for the restricted regressions.

In tables 6 and 7, I compare the Frisch and Marshallian elasticities respectively for rich and poor households. One sees across the regressions of table 6 the effect of excluding certain cross-prices in the estimation of elasticities and how, in the general model, the full set of elasticities can be estimated. Because of the interconnection between Frisch and Marshallian elasticities given above, greater generality affect all estimates.

I draw attention to a few elasticities to illustrate the model and the effect of the restrictions. Consider the Frisch elasticity of consumption with respect to interest factor. Note that because $r_t = p_{t+1}/(1+i_t)$, a *one percentage point increase* in i_t will result in an approximate one percent *decrease* in r_t. With time separability, the interest factor does not exert an independent effect conditional on μ as seen in the Frisch elasticities of table 6. When this channel is allowed for in the general regression, there appears a statistically significant and considerable effect which becomes stronger for poorer households. Going now to the same cells of table 7 which report Marshallian elasticities, one sees with the time separable regression that a one percentage point increase in interest rates lead to a 0.041% and 0.104% decrease in consumption for rich and poor households respectively. On the basis of t-scores, the latter is significant. On the other hand, when this effect is estimated with the general regression, one finds that a one percentage point increase in interest rates has the effect of *increasing* rich household consumption by 0.095% but *decreasing* poor household

Table 6. Frisch elasticities for rich and poor households.

	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
ε^F_{cp}	-0.156	0.026	-0.194	0.029	-0.422	0.119
ε^F_{cw}			-0.009	0.002	-0.002	0.008
ε^F_{cr}					-2.294	0.667
$\varepsilon^F_{c\mu}$	0.156	0.026	0.202	0.030	2.718	0.756
ε^F_{hp}			0.030	0.007	0.007	0.028
ε^F_{hw}	0.069	0.011	0.061	0.009	0.018	0.004
ε^F_{hr}					0.923	0.226
$\varepsilon^F_{h\mu}$	-0.069	0.011	-0.091	0.014	-0.947	0.246
ε^F_{Ap}					-0.762	0.220
ε^F_{Aw}					-0.089	0.022
ε^F_{Ar}	-1.105	0.185	-1.276	0.195	-8.601	2.271
$\varepsilon^F_{A\mu}$	1.105	0.185	1.276	0.195	9.452	2.511
	Poor Household					
ε^F_{cp}	-0.158	0.022	-0.215	0.033	-0.533	0.149
ε^F_{cw}			-0.013	0.004	-0.002	0.010
ε^F_{cr}					-2.896	0.808
$\varepsilon^F_{c\mu}$	0.158	0.022	0.227	0.036	3.431	0.917
ε^F_{hp}			0.037	0.010	0.007	0.028
ε^F_{hw}	0.055	0.007	0.049	0.006	0.018	0.004
ε^F_{hr}					0.926	0.213
$\varepsilon^F_{h\mu}$	-0.055	0.007	-0.085	0.013	-0.950	0.233
ε^F_{Ap}					-1.831	0.503
ε^F_{Aw}					-0.214	0.050
ε^F_{Ar}	-1.517	0.341	-1.980	0.382	-20.614	5.185
$\varepsilon^F_{A\mu}$	1.517	0.341	1.980	0.382	22.659	5.730

Table 7. Marshallian elasticities for rich and poor households.

	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
ε^M_{cp}	-0.191	0.027	-0.231	0.029	-0.274	0.079
ε^M_{cw}	0.013	0.002	0.006	0.002	0.047	0.007
ε^M_{cr}	0.014	0.024	0.041	0.027	-0.095	0.067
ε^M_{cW}	0.164	0.020	0.183	0.023	0.322	0.019
ε^M_{hp}	0.015	0.002	0.047	0.008	-0.045	0.022
ε^M_{hw}	0.063	0.011	0.054	0.009	0.001	0.004
ε^M_{hr}	-0.006	0.011	-0.019	0.012	0.156	0.020
ε^M_{hW}	-0.073	0.008	-0.082	0.010	-0.112	0.005
ε^M_{Ap}	-0.246	0.013	-0.235	0.015	-0.246	0.030
ε^M_{Aw}	0.091	0.002	0.094	0.002	0.081	0.003
ε^M_{Ar}	-1.005	0.008	-1.015	0.010	-0.953	0.021
ε^M_{AW}	1.160	0.013	1.156	0.014	1.118	0.015
	Poor Household					
ε^M_{cp}	-0.211	0.029	-0.268	0.037	-0.322	0.101
ε^M_{cw}	0.022	0.006	0.013	0.005	0.059	0.009
ε^M_{cr}	0.050	0.023	0.104	0.027	0.071	0.088
ε^M_{cW}	0.138	0.034	0.151	0.032	0.192	0.013
ε^M_{hp}	0.018	0.003	0.057	0.012	-0.052	0.022
ε^M_{hw}	0.047	0.007	0.039	0.006	0.001	0.004
ε^M_{hr}	-0.017	0.009	-0.039	0.011	0.104	0.020
ε^M_{hW}	-0.048	0.009	-0.057	0.010	-0.053	0.002
ε^M_{Ap}	-0.508	0.046	-0.462	0.052	-0.440	0.077
ε^M_{Aw}	0.215	0.007	0.222	0.008	0.194	0.009
ε^M_{Ar}	-1.036	0.018	-1.076	0.021	-1.021	0.052
ε^M_{AW}	1.330	0.049	1.316	0.051	1.268	0.040

consumption by 0.071%. The conclusions one draws about the impact of interest rates on consumption are reversed by the better fitting general regression. Other patterns that are noteworthy are that the positively sloped labor supply curve becomes appreciably more inelastic when estimated with the general regression and that the effect of increased wealth on labor supply differentially affects rich and poor households.

My general model has one other feature which is noteworthy: the ability to distinguish between household variation in μ arising from cross sectional variation and within household across time variation in μ_t arising from innovations in wages, interest rates or wealth. From above, μ depends on prices, wages, interest factors, exogenous income and starting wealth and its *variation* among households is a function of cross-sectional household variation of real wages, interest factors, exogenous income and starting wealth. Consequently, an increase of the wage in say 1985 is combined with wages in other years, interest factors, exogenous income and so forth before it results in an increase in μ.

However, I can also compute a corresponding elasticity of $\mu_{i,t}$ with respect to wages, interest factors or assets allowing for the impact of function f. Given my construction, $\mu_{i,t} = \delta^{(1988-t)} p_t / p_{1988} \exp(\theta_w (\tilde{w}_{i,t} - \tilde{w}_i) + \theta_r (\tilde{r}_{i,t} - \tilde{r}_i) + \theta_a (A_{i,t} - A_i))\mu_i$, the elasticity of $\mu_{i,t}$ with respect to its argument adds a component $\varepsilon^f_{fj} = j\theta_j$ where index j = w, r and A and here denotes real wage, real interest factor and asset respectively. The homogeneity of function f implies that the sum of elasticities with respect to p, w, r and W add to zero which allows for the recovery of the elasticity with respect to p. Table 8, divided into three panels, reports the elasticity of μ with respect to real wages, real interest factors and real wealth for rich and poor in the first and second panel. In the third panel, the elasticity of function f with respect to wages, interest factors and wealth is reported.

Table 8. Cross sectional μ and function f elasticities.

	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
$\varepsilon^{\mu}_{\mu w}$	0.082	0.013	0.073	0.010	0.018	0.003
$\varepsilon^{\mu}_{\mu r}$	0.090	0.144	0.205	0.114	0.809	0.025
$\varepsilon^{\mu}_{\mu A}$	1.050	0.172	0.906	0.135	0.118	0.031
	Poor Household					
$\varepsilon^{\mu}_{\mu w}$	0.141	0.029	0.112	0.019	0.018	0.002
$\varepsilon^{\mu}_{\mu r}$	0.317	0.142	0.456	0.095	0.865	0.011
$\varepsilon^{\mu}_{\mu A}$	0.876	0.174	0.665	0.113	0.056	0.014
	Elasticity of function f.					
ε^{f}_{fw}	0.940	0.041	0.801	0.047	0.082	0.018
ε^{f}_{fr}	0.317	0.437	0.424	0.383	0.924	0.053
ε^{f}_{fA}	1.059	0.173	1.024	0.149	0.159	0.040

A remarkable parallel exists between the elasticity of μ for rich households and the elasticity of f with respect to increases in assets. The elasticity of μ with respect to wealth is 1.050 in the demographic regression while the corresponding elasticity with respect to f is 1.059. In the general regression, both elasticities seem to decline by a similar magnitude. These figures are 0.118 and 0.159 respectively. This result is remarkable because these elasticities are derived from very different methodologies. The cross sectional variation of wealth for the sample gives me the estimate of this elasticity which is determined collectively by α, β and γ parameters. On the other hand, *growth* in the level of individual wealth over a five year span gives me the estimate of time series elasticity which is solely a function of the parameter θ_a. The consistency between these two estimates suggests this elasticity is estimated with some accuracy.

The elasticity of function f with respect to the interest factor across the regressions is rather interesting. Recall, the purpose of function f is to capture the evolution of marginal utility across time. Intertemporal optimization suggests the Euler equation, $\delta E_t(\lambda_{t+1}) = r_t \lambda_t / p_t$, where households equate the discounted expected future marginal utility of wealth saved with present marginal utility of wealth. Taking logs of this expression, it can be seen that the elasticity of μ_t with respect to r_t should be close to one.[17] The elasticity of function f with respect the interest factor is estimated at 0.317 and 0.424 in the demographic and time separable regressions respectively. On the other hand, in the general regression, this elasticity is estimated at 0.924 and is not statistically different from one. Additionally, the general regression estimates this elasticity with greater precision than the demographic and time separable regression judged by the standard errors associated with this estimate across the regressions. Because the general regression allow for the interest factor to enter into the structural equations explaining consumption and labor supply, it appears that removing this mechanism in the restrictive regressions biases the estimate of how μ_t varies with interest factor and, incorrectly, suggests households are poor intertemporal optimizers.

The consistency of cross-sectional and time series elasticity estimates with respect to wealth contrasts notably with that of wages. The elasticity of μ with respect to wages is 0.082 and 0.141 for rich and poor household respectively in the demographic regression. On

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[17] An expectation of Y given X, E(Y|X), is a function of X, not a function of Y. Thus, time t marginal utility will be a function of the time t interest factor, price, the discount factor and anything else that is part of the

the other hand, the elasticity of f with respect to wages is 0.940 in the same regression. In the general regression, the elasticity of μ with respect to wages is 0.018 for rich and poor households while the elasticity of f with respect to wages is 0.082, a figure 4.5 times greater. I account for this by the different construction of these elasticity estimates. The elasticity of μ with respect to wages combines many other variables into a single scalar measure. Although higher wages are likely to persist, no account of this is taken with the cross sectional measure of elasticity.

On the other hand, this restriction is not imposed in the time series estimate of elasticity. One can imagine, for example, an individual with low wages for the first 4 years of the sample with an unexpected increase in wages in the final year of the sample period. This individual may expect this new wage to persist and adjust consumption, work hours and savings target by a larger amount than a similarly situated individual whose wages will fall to the lower level in subsequent years.

If such an interpretation is correct, I can now compute long term Marshallian elasticities. I define long term Marshallian elasticities as that which not only recognize the dependence of individual specific prices and beginning wealth on μ, but also allows for the time t perturbation captured by function f. Using end of period assets as the example once more, $-A^* = \pi_r(p, w, r, f(p, w, r, W)\mu(p, w, r, W))$, this elasticity is defined as $\varepsilon^{LM}_{Aj} = \varepsilon^{F}_{Aj} + \varepsilon^{F}_{A\mu}(\varepsilon^{f}_{fj} + \varepsilon^{\mu}_{\mu j})$, where the superscript LM is used. The new term with the superscript f denotes the elasticity of function f with respect to any of its arguments. Such defined long term Marshallian elasticities are reported in table 9.

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information set of the household at time t. If r_t is not part of the information set of the household, then the

Table 9. Long term Marshallian Elasticities.

	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
ε_{cp}^{LM}	-0.555	0.115	-0.688	0.121	-3.496	0.745
ε_{cw}^{LM}	0.160	0.025	0.168	0.024	0.271	0.030
ε_{cr}^{LM}	0.065	0.079	0.129	0.089	2.472	0.693
ε_{cW}^{LM}	0.330	0.046	0.390	0.050	0.753	0.065
ε_{hp}^{LM}	0.176	0.040	0.252	0.047	1.078	0.222
ε_{hw}^{LM}	-0.002	0.003	-0.018	0.004	-0.077	0.006
ε_{hr}^{LM}	-0.029	0.035	-0.058	0.040	-0.738	0.214
ε_{hW}^{LM}	-0.146	0.018	-0.175	0.020	-0.263	0.017
ε_{Ap}^{LM}	-2.813	0.690	-3.116	0.677	-11.452	2.328
ε_{Aw}^{LM}	1.129	0.176	1.115	0.153	0.860	0.074
ε_{Ar}^{LM}	-0.646	0.501	-0.462	0.510	7.972	2.226
ε_{AW}^{LM}	2.330	0.241	2.463	0.231	2.620	0.151
	Poor Household					
ε_{cp}^{LM}	-0.494	0.085	-0.664	0.116	-4.118	0.887
ε_{cw}^{LM}	0.171	0.023	0.195	0.026	0.342	0.038
ε_{cr}^{LM}	0.101	0.074	0.202	0.094	3.311	0.844
ε_{cW}^{LM}	0.222	0.044	0.267	0.048	0.465	0.043
ε_{hp}^{LM}	0.117	0.026	0.206	0.043	0.999	0.207
ε_{hw}^{LM}	-0.004	0.003	-0.029	0.008	-0.078	0.005
ε_{hr}^{LM}	-0.035	0.027	-0.076	0.037	-0.793	0.203
ε_{hW}^{LM}	-0.078	0.011	-0.100	0.015	-0.129	0.008
ε_{Ap}^{LM}	-3.230	0.959	-3.918	1.035	-25.505	5.222
ε_{Aw}^{LM}	1.640	0.320	1.807	0.290	2.062	0.176
ε_{Ar}^{LM}	-0.544	0.692	-0.218	0.797	20.375	5.077
ε_{AW}^{LM}	2.134	0.226	2.329	0.230	3.068	0.191

elasticity will be exactly equal to one.

The most notable feature of these estimates when compared to those of table 7 giving short term Marshallian elasticities is the effect of wages. For example, the short term response of a rich household to a one percentage increase in wages on consumption, labor supply and wealth is 0.047%, 0.001% and 0.081% respectively but when this evaluated over the long term, the corresponding change is 0.271%, -0.077% and 0.860%. The labor supply curve to long term wages is now backward bending. Looking across regressions, the slope of the more restrictive functional forms understates the extent of the backward bend in labor.

CONCLUSION

My thesis sought to examine if duality techniques and greater generality can be profitably employed in the modeling of dynamic household choice. My model treats intertemporal choice as a three good problem with choice variables consumption, labor supply and savings subject to a budget constraint- a treatment very similar to techniques used by demand modelers. In doing so, the invention of a new functional form was required which allows for the inversion of the budget constraint to determine an explicit expression for the unobserved marginal utility of income. This is, arguably, the most substantive contribution of this paper.

In addition to meeting this necessary requirement, my functional form has appealing properties. My functional form is globally regular, rank 3 and is derived from a Laurent series approximation rather than the Taylor series approximation often used in flexible functional forms. Empirically, it can be seen that the model fits the data well.

With this new model, I tested two commonly maintained hypotheses and decisively rejected both consumption-labor additivity and time separability. These restrictions on the primal maximization problem amount- in the dual- to an imposition of zero cross-price Frisch elasticities which I find are restrictions that should be rejected. Removing these cross-price restrictions changes the entire set of estimated Frisch and Marshallian elasticities. Additionally, important intertemporal parameters such as the rate of time preference and the time series elasticity of the marginal utility of income with respect to changes in wage, interest factor and wealth are more precisely measured by my more general regression.

These results cast doubt on contemporary elasticity estimates made using the more restrictive forms that have become economic lore. For example, the common assertion that consumption falls when interest rates rise holds only for households with low wealth. An income effect reverses this conclusion for rich households. Also, the common assertion that labor supply is positively sloped was found only to apply to transitory wage increases. Should the wage increase be permanent or at least persistent, the wealth effect of higher wages causes labor supply to contract.

However, more than the particulars of my findings, what I hope I offer to the profession is a fruitful approach that opens new areas in economics. To this end, I offer what I consider worthwhile extensions of my framework. Some of these are merely technical refinements but others have the potential to affect other fields of economics.

On a technical level, I estimated preferences for single headed households using what amounts to a single cross-section. Although the data were in panel format, it was necessary to have beginning and end wealth to determine consumption. Thus, one does not have true “within” and “between" errors analogous to variance component models. To do this, a third wealth supplement is required. This would allow analysis of household behavior across time and in cross-section. This was not done in this paper because, at the time of writing, final release PSID data for the next wealth supplement in 1994 was unavailable. The use of early release data would have allowed for the analysis to include wealth from the 1994, 1999 and 2001 waves of the PSID, however I relied on numerous constructed variables unavailable in the early release files, for example federal taxes.

Another important development would be extending this to couples. The number of married and joint households is 3 times larger that single headed households which allows for a better analysis simply from having more data alone. The household intertemporal problem can be conceived of as a four good problem: how much to consume, how much to save, and the labor supply of the head and the spouse as separate choices. It would be interesting to allow for the interaction between head and spouse labor supply.

Another extension is to model the demand for particular classes of assets. I aggregated all forms of wealth and derived a composite return of this wealth. However, given that the PSID has the individual return of many assets, it would seem possible to treat these as distinct goods, each with its own price. This would then allow the analysis of substitutability or complementarity of the different assets. Another extension or refinement would be to further generalize the functional form. For example, the literature on precautionary savings suggests that income volatility increases the demand for wealth. This analysis can be captured in my framework by incorporating income volatility measures in the sub-function π^{γ}. In general, what was surprising to me was that every generalization I attempted proved to be statistically significant, suggesting that the search for even greater generality and improved fit has not been exhausted. While this is true, it is also true that estimation at times proved to be a substantial challenge. Yet another refinement would be to improve the efficiency of my estimation procedure.

Because of the central importance of household choice in much of economics, these results have wide-ranging implications for other fields in economics and for public policy. These results should be incorporated into business cycle theories where modeling the dynamic behavior of households is of central importance. Another application is in the field of social welfare functions. This model identifies the marginal utility of income which occupies a central place in analyses that consider the redistribution of income. A further application is in the area of tax incidence. A fundamental policy choice is the balance of taxes on consumption (sales and value added taxes), labor earnings (social security, payroll and income taxes) and wealth (corporate income, property and wealth taxes) and this model informs on each of these elasticities. Another application is in general equilibrium models where the flows to and from firms are matched with the flows from and to households. The supply of consumption by firms to household and the supply of labor by households to firms are obvious, but it would also seem possible that the wealth demand of households could be translated into the capital requirement of firms.

I offer my model in the hope economists find this fertile ground.

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APPENDIX

```
                         TSP Version 4.5
                  (12/18/03) Windows32    20MB
              Copyright (C) 2003 TSP International
                       ALL RIGHTS RESERVED
                        04/03/04 10:43AM
        In case of questions or problems, see your local TSP
        consultant or send a description of the problem and the
        associated TSP output to:
                         TSP International
                          P.O. Box 61015
                       Palo Alto, CA 94306
                               USA
         PROGRAM
COMMAND  ***************************************************************
|      1  options memory=20, limwarn=5;
|      2
|      2  ?*--------------------------------------------------------------*
|      2  ?|                                                              ?|
|      2  ?|     Name: G4    Final Model INCLUDE INVERSIONS               ?|
|      2  ?|     Purpose: This use the budget constraint of rank three    ?|
|      2  ?| pi = homogeneous one function b's without mu +               ?|
|      2  ?|    + terms with mu*A(p) + terms C(p)/mu                      ?|
|      2  ?|                                                              ?|
|      2  ?|   Date: 4/3/04                                              ?|
|      2  ?|                                                              ?|
|      2  ?*--------------------------------------------------------------;
|      2
|      2  ?* Read the data file;
|      2
|      2  read (file='singl.xls',format=excel) id85
|      2  hdwkhr85 nodep85 age85 hdwage85 hdwkhu85
|      2  hdwkhr86 nodep86 age86 hdwage86 hdwkhu86
|      2  hdwkhr87 nodep87 age87 hdwage87 hdwkhu87
|      2  hdwkhr88 nodep88 age88 hdwage88 hdwkhu88
|      2  hdwkhr89 nodep89 age89 sexhd hdwage89 capgain hdwkhu89
|      2  wun85 wun86 wun87 wun88 wun89
|      2  wth84 wth85 wth86 wth87 wth88 wth89 i85 i86 i87 i88 i89
|      2  y85 y86 y87 y88 y89 m endowm virtinc
|      2  s1 s2 s3 s4 s5 s6 s7 su3 su4 su5 su6 su7
|      2  ss1 ss2 ss3 ss4 ss5 ss6 ss7;
|      3
|      3  ???????  Set up various demographic variables????????????
|      3  ???????? Normalized with means where appropriate?????????
|      3  ???????? and normalized wages and expenditures??????????
|      3  ??????? Setting up price constants ????????????????????
|      3
|      3  set p85=103.9/118.3; set p86=107.6/118.3; set p87=109.6/118.3;
|      6  set p88=113.6/118.3; set p89=1;
|      8  set rr85=1;
|      9  rr86=1/(1+i85); rr87=rr86/(1+i86);  rr88=rr87/(1+i87);
  rr89=rr88/(1+i88);
|     13
|     13  r85=p86/(1+i85); r86=p87/(1+i86); r87=p88/(1+i87);
|     16  r88=p89/(1+i88); r89=124.0/118.3/(1+i89);
|     18
|     18  dot 5-9;
|     19  agen8.=(age8.- 45); agen8.s=agen8.^2; agen8.c=agen8.^3;
|     22  nodep8.n=nodep8.  - 1.3314;
```

```
23  w8.=(wun8.);
24  iwth8.=1/wth8. - 0.000061052;
25  enddot;
26
26  stddevi=((
26  ( (i85^2 + i86^2 + i87^2 + i88^2 + i89^2) -
26     (i85 + i86 + i87 + i88 + i89)^2 / 5)  / 4   )^0.5 -
0.04507723)/0.058702;
27
27  reldevi=((
27  ( (i85^2 + i86^2 + i87^2 + i88^2 + i89^2) -
27     (i85 + i86 + i87 + i88 + i89)^2 / 5)  / 4   )^0.5   /
27  (i85 + i86 + i87 + i88 + i89 + 5)* 5 - 0.03770829)/0.044247;
28
28  stddevw=((
28  ( (w85^2 + w86^2 + w87^2 + w88^2 + w89^2) -
28     (w85 + w86 + w87 + w88 + w89)^2 / 5)  / 4   )^0.5 -
0.31532218)/0.52813;
29
29  reldevw=((
29  ( (w85^2 + w86^2 + w87^2 + w88^2 + w89^2) -
29     (w85 + w86 + w87 + w88 + w89)^2 / 5)  / 4   )^0.5    /
29  (w85 + w86 + w87 + w88 + w89 + 0.0001)* 5 - 0.3537256)/0.55709;
30
30  ??????????????????????????????????????????????????????????????
30  ????????????Report Sample characteristics ????????????????
30  ??????????????????????????????????????????????????????????????
30  goto 50;
31  50 msd (terse) id85
31  hdwkhr85 nodep85 age85 hdwage85 hdwkhu85
31  hdwkhr86 nodep86 age86 hdwage86 hdwkhu86
31  hdwkhr87 nodep87 age87 hdwage87 hdwkhu87
31  hdwkhr88 nodep88 age88 hdwage88 hdwkhu88
31  hdwkhr89 nodep89 age89 sexhd hdwage89 capgain hdwkhu89
31  wun85 wun86 wun87 wun88 wun89
31  wth84 wth85 wth86 wth87 wth88 wth89 i85 i86 i87 i88 i89
31  y85 y86 y87 y88 y89 m endowm virtinc
31  s1 s2 s3 s4 s5 s6 s7 su3 su4 su5 su6 su7
31  ss1 ss2 ss3 ss4 ss5 ss6 ss7
31  rr86-rr89 agen85-agen89 nodep85n nodep86n
31  nodep87n nodep88n nodep89n
31  stddevi reldevi stddevw reldevw iwth85-iwth89
31  ;
32
32  ?????????????????????????????????????????????????????????????
32  ??????????????????Set up parameter bank?????????????????
32  ?????????????????????????????????????????????????????????????
32  const
32  fa11 1 fa22 1 fa33 1
32  fa12 0 fa13 0 fa23 0
32  fb12 0 fb13 0 fb23 0
32  fc11 130  fc22 1 fc33 1
32  ;
33  set eps=0;
34
34  frml eqb389 b389=
34  (b330*(2-sexhd) + b331*(2-sexhd)*agen89 + b332*(2-sexhd)*agen89s +
34   b333*(2-sexhd)*agen89c + b334*(2-sexhd)*nodep89n +
34   b335*(sexhd-1) + b336*(sexhd-1)*agen89 + b337*(sexhd-1)*agen89s +
34   b338*(sexhd-1)*agen89c + b339*(sexhd-1)*nodep89n);
35
```

```
35  dot 5-9;
36  frml eqb18. b18.=
36  (b110*(2-sexhd) + b111*(2-sexhd)*agen8. + b112*(2-sexhd)*agen8.s +
36   b113*(2-sexhd)*agen8.c + b114*(2-sexhd)*nodep8.n +
36   b115*(sexhd-1) + b116*(sexhd-1)*agen8. + b117*(sexhd-1)*agen8.s +
36   b118*(sexhd-1)*agen8.c + b119*(sexhd-1)*nodep8.n);
37
37  frml eqb28. b28.=
37  (b220*(2-sexhd) + b221*(2-sexhd)*agen8. + b222*(2-sexhd)*agen8.s +
37   b223*(2-sexhd)*agen8.c + b224*(2-sexhd)*nodep8.n +
37   b225*(sexhd-1) + b226*(sexhd-1)*agen8. + b227*(sexhd-1)*agen8.s +
37   b228*(sexhd-1)*agen8.c + b229*(sexhd-1)*nodep8.n);
38
38  frml eqop8. op8. =  -b12*(p8.*w8.)^0.5 - b13*p8.*(r8.)^0.5 ;
39  frml eqow8. ow8. =  -b12*(p8.*w8.)^0.5 - b23*(w8.*p8.*r8.)^0.5 ;
40  enddot;
41
41  frml eqor89 or89 =  -b13*p89*(r89)^0.5 - b23*(w89*p89*r89)^0.5 ;
42
42  ???????? Sign Restrictions on Parameters ??????????
42  ??????????????????????????????????????????????????
42  frml eqa11 a11= ra33*((ra11>=eps*fa11)*(ra11-eps*fa11) + eps*fa11);
43  frml eqa22 a22= ra33*((ra22>=eps*fa22)*(ra22-eps*fa22) + eps*fa22);
44
44  frml eqa33 a33= ra33;
45  frml eqa12 a12= ra33*((ra12>=eps*fa12)*(ra12-eps*fa12) + eps*fa12);
46  frml eqa13 a13= ra33*((ra13>=eps*fa13)*(ra13-eps*fa13) + eps*fa13);
47  frml eqa23 a23= ra33*((ra23>=eps*fa23)*(ra23-eps*fa23) + eps*fa23);
48
48  frml eqb12 b12= (rb12>=eps*fb12)*(rb12-eps*fb12) + eps*fb12;
49  frml eqb13 b13= (rb13>=eps*fb13)*(rb13-eps*fb13) + eps*fb13;
50  frml eqb23 b23= (rb23>=eps*fb23)*(rb23-eps*fb23) + eps*fb23;
51
51  frml eqc11 c11 =((rc11>=eps*fc11)*(rc11-eps*fc11) + eps*fc11)^2;
52  frml eqc12 c12 =((rc11>=eps*fc11)*(rc11-eps*fc11) + eps*fc11)*rc12;
53  frml eqc13 c13 =((rc11>=eps*fc11)*(rc11-eps*fc11) + eps*fc11)*rc13;
54  frml eqc22 c22 =  rc12^2 +
54                  ((rc22>=eps*fc22)*(rc22-eps*fc22) + eps*fc22)^2;
55  frml eqc23 c23 =  rc12*rc13 +
55                  ((rc22>=eps*fc22)*(rc22-eps*fc22) + eps*fc22)*rc23;
56  frml eqc33 c33 =  rc13^2 + rc23^2 +
56                  ((rc33>=eps*fc33)*(rc33-eps*fc33) + eps*fc33)^2;
57
57  ??????????????????????????????????????????????????????????????
57  ??????????????????  Quadratic MU Calculation ??????????????????
57  frml eqaa aa=    {
57  [-a11  - a13*p85/(p85+aa13*r85)  -a12
57   -a22  - a23*w85/(w85+aa23*r85)     ]*rr85/ra85 +
57
57  [-a11  - a13*p86/(p86+aa13*r86)  -a12
57   -a22  - a23*w86/(w86+aa23*r86)     ]*rr86/ra86 +
57
57
57  [-a11  - a13*p87/(p87+aa13*r87)  -a12
57   -a22  - a23*w87/(w87+aa23*r87)     ]*rr87/ra87 +
57
57  [-a11  - a13*p88/(p88+aa13*r88)  -a12
57   -a22  - a23*w88/(w88+aa23*r88)     ]*rr88/ra88 +
57
57  [-a11 -a12 -a13 -a22 -a23 -a33      ]*rr89/ra89 };
58
```

```
58
58
58  frml eqbb bb=   endowm + {
58  [ b185*p85 + b285*w85   +  op85 +  ow85 ]  +
58
58  [ b186*p86 + b286*w86   +  op86 +  ow86 ]*rr86 +
58
58  [ b187*p87 + b287*w87   +  op87 +  ow87 ]*rr87 +
58
58  [ b188*p88 + b288*w88   +  op88 +  ow88 ]*rr88 +
58
58  [ b189*p89 + b289*w89 + b389*r89  +  op89 +  ow89 +  or89 ]*rr89 };
59
59  eqsub eqbb
59  eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
59  eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
59  eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
60
60
60  frml eqcc cc=    {
60  [c11*p85^2  +   2*c12*p85*w85 +   c13*p85*r85 +
60   c22*w85^2  +     c23*w85*r85]*rr85*ra85  +
60
60  [c11*p86^2  +   2*c12*p86*w86 +   c13*p86*r86 +
60   c22*w86^2  +     c23*w86*r86]*rr86*ra86 +
60
60  [c11*p87^2  +   2*c12*p87*w87 +   c13*p87*r87 +
60   c22*w87^2  +     c23*w87*r87]*rr87*ra87 +
60
60  [c11*p88^2  +   2*c12*p88*w88 +   c13*p88*r88 +
60   c22*w88^2  +     c23*w88*r88]*rr88*ra88 +
60
60  [c11*p89^2  +   2*c12*p89*w89 + 2*c13*p89*r89 +
60   c22*w89^2  +   2*c23*w89*r89 +  c33*r89^2  ]*rr89*ra89 };
61
61
61  ? frml eqlama lama=    ( - bb )/aa/( bb > 0 )   ;
61  frml eqmu  mu =  ( - bb - (bb^2  -  4*aa*cc)^0.5)/aa/2   ;
62  frml eqlam lam=  ( - bb + (bb^2  -  4*aa*cc)^0.5)/cc/2   ;
63
63
63
63  ??????????????????????????????????????????????????????
63  ???????? Estimated Expenditures ????????????????????????
63  ??????????????????????????????????????????????????????
63
63  frml eqae89 ae89=[   b389*r89  +   or89  +
63  (-a33  -  a13*aa13*r89/(p89+aa13*r89)  -
63           a23*aa23*r89/(w89+aa23*r89)      )*mu/ra89
63   +(c33*r89^2  +  c13*p89*r89 + c23*w89*r89)*lam*ra89  ]*rr89;
64
64
64  dot 5-9;
65
65  frml eqce8. ce8.=[   b18.*p8.  +   op8. +
65  (-a11  -  a12*p8./(p8.+aa12*w8.) -
65           a13*p8./(p8.+aa13*r8.)            )*mu/ra8.
65   +(c11*p8.^2  +   c12*p8.*w8. + c13*p8.*r8.)*lam*ra8. ]*rr8.;
66
66  frml eqwe8. we8.=[   b28.*w8. +   ow8.   +
66  (-a22 - a12*aa12*w8./(p8.+aa12*w8.)/((p8.+aa12*w8.)>0) -
```

```
  66                a23*w8./(w8.+aa23*r8.)      )*mu/ra8.
  66   +(c22*w8.^2  +    c12*p8.*w8. + c23*w8.*r8.)*lam*ra8. ]*rr8.;
  67
  67  enddot;
  68
  68  ?????????? This area sets up expectations ????????????
  68  ?????????????????????????????????????????????????????????????/
  68  ? branch=1 is the naive model
  68  ? branch=2 is the perfect foresight model
  68  ? branch=3 is the rational expectations model
  68
  68  SET BRANCH = 4;
  69
  69  If (branch = 1); then; goto 100;
  72
  72  If (branch = 2); then; goto 120;
  75  If (branch = 3); then; goto 140;
  78  If (branch = 4); then; goto 160;
  81
  81  100 title " Naive Model Branch is 1111";
  82  frml eqra85 ra85 = 1;
  83  frml eqra86 ra86 = p85/p86;
  84  frml eqra87 ra87 = p85/p87;
  85
  85  frml eqra88 ra88 = p85/p88;
  86  frml eqra89 ra89 = p85/p89;
  87
  87  goto 200;
  88
  88  120 title " Perfect Foresight Model Branch is 2222";
  89  frml eqra85 ra85 = 1;
  90  frml eqra86 ra86 = (r85)/discf;
  91  frml eqra87 ra87 = (r85*r86)/discf^2;
  92  frml eqra88 ra88 = (r85*r86*r87)/discf^3;
  93  frml eqra89 ra89 = (r85*r86*r87*r88)/discf^4;
  94
  94  param discf 0.98;
  95
  95
  95  goto 200;
  96
  96  140 title " Rational Expectations Model Branch is 333";
  97  frml eqra85 ra85 = (1 +
ew*(4*w85/p85-w86/p86-w87/p87-w88/p88-w89/p89)/5 +
  97  er*(4*r85/p85-r86/p86-r87/p87-r88/p88-r89/p89)/5 +
ea*(wth85/p85-wth87/p87)  );
  98  frml eqra86 ra86 = (1 +
ew*(-w85/p85+4*w86/p86-w87/p87-w88/p88-w89/p89)/5 +
  98  er*(-r85/p85+4*r86/p86-r87/p87-r88/p88-r89/p89)/5 +
ea*(wth86/p86-wth87/p87)  )*p85/p86;
  99  frml eqra87 ra87 = (1 +
ew*(-w85/p85-w86/p86+4*w87/p87-w88/p88-w89/p89)/5 +
  99  er*(-r85/p85-r86/p86+4*r87/p87-r88/p88-r89/p89)/5    )*p85/p87;
 100  frml eqra88 ra88 = (1 +
 ew*(-w85/p85-w86/p86-w87/p87+4*w88/p88-w89/p89)/5 +
 100  er*(-r85/p85-r86/p86-r87/p87+4*r88/p88-r89/p89)/5 +
 ea*(wth88/p88-wth87/p87)  )*p85/p88;
 101  frml eqra89 ra89 = (1 +
 ew*(-w85/p85-w86/p86-w87/p87-w88/p88+4*w89/p89)/5 +
 101  er*(-r85/p85-r86/p86-r87/p87-r88/p88+4*r89/p89)/5 +
 ea*(wth89/p89-wth87/p87)  )*p85/p89;
```

```
param ew 0 er 0 ea 0;
160 title " Rational Expectations Model Branch is 444";
frml eqra85 ra85 =
exp(4*et+ew*(4*w85/p85-w86/p86-w87/p87-w88/p88-w89/p89)/5 +
er*(4*r85/p85-r86/p86-r87/p87-r88/p88-r89/p89)/5 +
ea*(wth85/p85-wth87/p87)  );
frml eqra86 ra86 =
exp(3*et+ew*(-w85/p85+4*w86/p86-w87/p87-w88/p88-w89/p89)/5 +
er*(-r85/p85+4*r86/p86-r87/p87-r88/p88-r89/p89)/5 +
ea*(wth86/p86-wth87/p87)  )*p85/p86;
frml eqra87 ra87 =
exp(2*et+ew*(-w85/p85-w86/p86+4*w87/p87-w88/p88-w89/p89)/5 +
er*(-r85/p85-r86/p86+4*r87/p87-r88/p88-r89/p89)/5   )*p85/p87;
frml eqra88 ra88 =
exp(et+ew*(-w85/p85-w86/p86-w87/p87+4*w88/p88-w89/p89)/5 +
er*(-r85/p85-r86/p86-r87/p87+4*r88/p88-r89/p89)/5 +
ea*(wth88/p88-wth87/p87)  )*p85/p88;
frml eqra89 ra89 =
exp(ew*(-w85/p85-w86/p86-w87/p87-w88/p88+4*w89/p89)/5 +
er*(-r85/p85-r86/p86-r87/p87-r88/p88+4*r89/p89)/5 +
ea*(wth89/p89-wth87/p87)  )*p85/p89;
param ew 0 er 0 ea 0 et 0;
?????????? Expenditure Share Estimating Equations????????????
?????????????????????????????????????????????????????????????/
200 frml eqs1 ss1=(-(ae89));
frml eqs2 ss2=(-(ce85+ce86+ce87+ce88+ce89));
frml eqs3 ss3=we85;
frml eqs4 ss4=we86;
frml eqs5 ss5=we87;
frml eqs6 ss6=we88;
frml eqs7 ss7=we89;
eqsub eqs1 eqae89 eqce85-eqce89 eqwe85-eqwe89 eqlam eqmu eqaa eqbb
eqcc eqra85-eqra89
eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
eqsub eqs2 eqae89 eqce85-eqce89 eqwe85-eqwe89 eqlam eqmu eqaa eqbb
eqcc eqra85-eqra89
eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
eqsub eqs3 eqae89 eqce85-eqce89 eqwe85-eqwe89 eqlam eqmu eqaa eqbb
eqcc eqra85-eqra89
eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389

eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
eqsub eqs4 eqae89 eqce85-eqce89 eqwe85-eqwe89 eqlam eqmu eqaa eqbb
eqcc eqra85-eqra89
eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
eqsub eqs5 eqae89 eqce85-eqce89 eqwe85-eqwe89 eqlam eqmu eqaa eqbb
eqcc eqra85-eqra89
eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
```

```
|    121  eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
|    122  eqsub eqs6 eqae89 eqce85-eqce89 eqwe85-eqwe89 eqlam eqmu eqaa eqbb
     eqcc eqra85-eqra89
|    122  eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
|    122  eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
|    122  eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
|    123  eqsub eqs7 eqae89 eqce85-eqce89 eqwe85-eqwe89 eqlam eqmu eqaa eqbb
     eqcc eqra85-eqra89
|    123  eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
|    123  eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
|    123  eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
|    124
|    124
|    124  ?????????????????Parameter List and Initialization ????????????
|    124  ????????????????????????????????????????????????????????????????
|    124
|    124
|    124  const   ra33 1
|    124  aa12 1 aa13 1 aa23 1 ;
|    125
|    125  const
|    125
|    125  b220 0 b330 0
|    125  b225 0 b335 0
|    125
|    125  b111 0 b112 0 b113 0 b114 0
|    125  b116 0 b117 0 b118 0 b119 0
|    125  b221 0 b222 0 b223 0 b224 0
|    125  b226 0 b227 0 b228 0 b229 0
|    125  b331 0 b332 0 b333 0 b334 0
|    125  b336 0 b337 0 b338 0 b339 0
|    125
|    125  ra22 1   ra33 1
|    125  ra12 0 ra13 0 ra23 0
|    125
|    125  rb12 0 rb13 0 rb23 0
|    125
|    125  rc11 11.557508 rc22 0.2  rc33 1
|    125  rc12 0    rc13 0     rc23 0
|    125  ;
|    126
|    126  set excel=0; ? This is the switch that creates excel output
|    127
|    127  ???????????????????????????????????????????????????????????????
|    127  ?????????????? Stochastic Effects Model ?????????????????????
|    127  ??????????????? Additivity ???????????????????????????????????
|    127  ???????????????????????????????????????????????????????????????
|    127   ? Number of observations = 525      Log likelihood = -29614.0
|    127   ? Below is new regression (3/27/04) above is old
|    127   ?  Number of observations = 525      Log likelihood = -29612.3
|    127   ?  Number of observations = 525      Log likelihood = -29610.8
|    127   ?  Number of observations = 525      Log likelihood = -29610.7
|    127   ?  Number of observations = 525      Log likelihood = -29610.7
|    127   ?  RC22        2223.53       163.201        13.6244       [.000]
|    127   ?  RC11        28710.7       4016.03        7.14903       [.000]
|    127   ?  RC33        29596.9       5010.52        5.90696       [.000]
|    127   ?  RA22        .257340E-02   .238458E-03    10.7918       [.000]
|    127   ?  RA11        -.290798E-03  0.             0.            [1.00]
|    127   ?  B330        285239.       37140.6        7.67997       [.000]
|    127   ?  B335        -9581.63      14231.9        -.673250      [.501]
|    127   ?  EW          -.176508      .388387E-02    -45.4466      [.000]
```

```
127  ?  ER          -.622931       .216430        -2.87820       [.004]
127  ?  EA          -.102695E-05   .879001E-06    -1.16832       [.243]
127  ?  ET          -.013260       .640921E-02    -2.06895       [.039]
127  ?  B220        1713.87        17.2838        99.1600        [.000]
127  ?  B225        859.829        16.3234        52.6746        [.000]
127  ?  B110        -20239.7       727.455        -27.8226       [.000]
127  ?  B115        -30952.8       1800.62        -17.1901       [.000]
127
127  set skip = 1;
128  const
128  fa11 1 fa22 0.001 fa33 1
128  fa12 0 fa13 0 fa23 0
128  fb12 0 fb13 0 fb23 0
128  fc11 11  fc22 1 fc33 1
128
128  ;
129  set eps=0;
130
130  Title ' **** Basic Model *****';
131
131  ;
131  param
131   RC22       2223.53
131   RC11       28710.7
131   RC33       29596.9
131   RA22       .257340E-02
131   RA11       0
131   B330       285239.
131   B335       -9581.63
131   EW         -.176508
131   ER         -.622931
131   EA         -.102695E-05
131   ET         -.013260
131   B220       1713.87
131   B225       859.829
131   B110       -20239.7
131   B115       -30952.8
131  ;
132
132    estimate;
133
133
133  ???????????????  Demographic Info ??????????????????????????????
133  ????????????????????????????????????????????????????????????????
133   ?  Number of observations = 525      Log likelihood = -29370.5
133   ? Below is new regression (3/27/04) above is old
133   ?  Number of observations = 525      Log likelihood = -29375.4
133   ?  below uses et parameter (local maxima might be a problem)
133   ?  Number of observations = 525      Log likelihood = -29363.6
133   ?  RC22       756.481        82.8176        9.13430        [.000]
133   ?  RC11       9634.18        1135.30        8.48602        [.000]
133   ?  RC33       21941.3        2215.69        9.90266        [.000]
133   ?  RA22       .460416E-02    .563831E-03    8.16584        [.000]
133   ?  RA11       .033174        .592790E-02    5.59623        [.000]
133   ?  B330       -1183.62       17664.7        -.067005       [.947]
133   ?  B331       -4521.38       776.344        -5.82394       [.000]
133   ?  B332       -162.767       53.8131        -3.02468       [.002]
133   ?  B333       4.62436        1.55732        2.96943        [.003]
133   ?  B334       15040.8        18891.3        .796176        [.426]
133   ?  B335       -54760.2       6839.23        -8.00677       [.000]
133   ?  B336       -2484.51       444.861        -5.58492       [.000]
```

```
133   ? B337      -117.707      30.8195       -3.81923     [.000]
133   ? B338      2.90496       .688999       4.21621      [.000]
133   ? B339      -6646.11      4575.30       -1.45261     [.146]
133   ? EW        -.229254      .010117       -22.6600     [.000]
133   ? ER        -.348889      .479996       -.726857     [.467]
133   ? EA        -.105960E-04  .173164E-05   -6.11906     [.000]
133   ? ET        -.068935      .015723       -4.38435     [.000]
133   ? B220      1859.17       34.4218       54.0114      [.000]
133   ? B221      -34.7786      1.99068       -17.4707     [.000]
133   ? B222      -1.44080      .136699       -10.5400     [.000]
133   ? B223      .021480       .544818E-02   3.94265      [.000]
133   ? B224      -3.63520      12.9628       -.280434     [.779]
133   ? B225      1408.02       30.6263       45.9744      [.000]
133   ? B226      -20.3281      2.06141       -9.86126     [.000]
133   ? B227      -.855229      .123272       -6.93776     [.000]
133   ? B228      .012911       .556962E-02   2.31814      [.020]
133   ? B229      39.5846       15.3558       2.57783      [.010]
133   ? B110      -23378.2      1209.48       -19.3292     [.000]
133   ? B115      -25630.9      1125.42       -22.7745     [.000]
133
133
133  set skip = 1;
134  const
134  fa11 1 fa22 0.001 fa33 1
134  fa12 0 fa13 0 fa23 0
134  fb12 0 fb13 0 fb23 0
134  fc11 11  fc22 1 fc33 1;
135  set eps=0;
136
136
136  Title ' **** Demo Model *****';
137  param
137   RC22       756.481
137   RC11       9634.18
137   RC33       21941.3
137   RA22       .460416E-02
137   RA11       .033174
137   B330       -1183.62
137   B331       -4521.38
137   B332       -162.767
137   B333       4.62436
137   B334       15040.8
137   B335       -54760.2
137   B336       -2484.51
137   B337       -117.707
137   B338       2.90496
137   B339       -6646.11
137   EW         -.229254
137   ER         -.348889
137   EA         -.105960E-04
137   ET         -.068935
137   B220       1859.17
137   B221       -34.7786
137   B222       -1.44080
137   B223       .021480
137   B224       -3.63520
137   B225       1408.02
137   B226       -20.3281
137   B227       -.855229
137   B228       .012911
137   B229       39.5846
```

```
137  B110       -23378.2
137  B115       -25630.9
137  ;
138
138   estimate;
139
139
139  ????????????????????????????????????????????????????????????????????
139  ???????????????   Now time separability?????????????????????????
139  ????????????????????????????????????????????????????????????????????
139   ?  Number of observations = 525      Log likelihood = -29364.6
139   ? Below is new regession (3/27/04) above is old.
139   ?  Number of observations = 525      Log likelihood = -29371.2
139   ?  Number of observations = 525      Log likelihood = -29371.2
139   ?  Number of observations = 525      Log likelihood = -29354.6
139   ?  Number of observations = 525      Log likelihood = -29354.6
139   ?  RC12        247.215        67.7260        3.65023        [.000]
139   ?  RC22        723.523        84.3078        8.58192        [.000]
139   ?  RC11        13034.9        1787.51        7.29218        [.000]
139   ?  RC33        29836.0        3641.60        8.19311        [.000]
139   ?  RA22        -.404169E-03   0.             0.             [1.00]
139   ?  RA12        .563513E-02    .700763E-03    8.04142        [.000]
139   ?  RA11        .032811        .670305E-02    4.89488        [.000]
139   ?  B330        7277.14        18705.7        .389033        [.697]
139   ?  B331        -4354.15       853.372        -5.10229       [.000]
139   ?  B332        -191.485       56.7390        -3.37483       [.001]
139   ?  B333        5.07762        1.66046        3.05797        [.002]
139   ?  B334        15779.9        15619.6        1.01027        [.312]
139   ?  B335        -59582.7       8261.09        -7.21246       [.000]
139   ?  B336        -2741.34       482.861        -5.67728       [.000]
139   ?  B337        -115.145       32.4878        -3.54427       [.000]
139   ?  B338        2.90590        .720636        4.03242        [.000]
139   ?  B339        -5367.69       4905.89        -1.09413       [.274]
139   ?  EW          -.194917       .011538        -16.8938       [.000]
139   ?  ER          -.465212       .420025        -1.10758       [.268]
139   ?  EA          -.102237E-04   .148672E-05    -6.87666       [.000]
139   ?  ET          -.076539       .015315        -4.99772       [.000]
139   ?  RB12        -266.206       0.             0.             [1.00]
139   ?  B220        1856.04        36.2815        51.1566        [.000]
139   ?  B221        -33.7890       1.97426        -17.1148       [.000]
139   ?  B222        -1.46942       .137090        -10.7186       [.000]
139   ?  B223        .020672        .525700E-02    3.93235        [.000]
139   ?  B224        -.850843       12.0341        -.070703       [.944]
139   ?  B225        1384.76        32.9775        41.9912        [.000]
139   ?  B226        -20.4162       1.98869        -10.2662       [.000]
139   ?  B227        -.823469       .121763        -6.76287       [.000]
139   ?  B228        .010089        .538940E-02    1.87192        [.061]
139   ?  B229        38.5797        15.4789        2.49240        [.013]
139   ?  B110        -24126.5       1470.85        -16.4031       [.000]
139   ?  B115        -27545.3       1418.73        -19.4154       [.000]
139
139  set skip = 1;
140  const
140  fa11 1 fa22 0.001 fa33 17
140  fa12 0.001 fa13 0 fa23 0
140  fb12 0.001 fb13 0 fb23 0
140  fc11 11  fc22 1 fc33 1;
141  set eps=0;
142
142  ? held constant because of corner
142  const ra22 0;
```

```
143
143  Title ' **** T/S Model *****';
144  param
144   RC12       247.215
144   RC22       723.523
144   RC11       13034.9
144   RC33       29836.0
144   RA12       .563513E-02
144   RA11       .032811
144   B330       7277.14
144   B331       -4354.15
144   B332       -191.485
144   B333       5.07762
144   B334       15779.9
144   B335       -59582.7
144   B336       -2741.34
144   B337       -115.145
144   B338       2.90590
144   B339       -5367.69
144   EW         -.194917
144   ER         -.465212
144   EA         -.102237E-04
144   ET         -.076539
144   RB12       -266.206
144   B220       1856.04
144   B221       -33.7890
144   B222       -1.46942
144   B223       .020672
144   B224       -.850843
144   B225       1384.76
144   B226       -20.4162
144   B227       -.823469
144   B228       .010089
144   B229       38.5797
144   B110       -24126.5
144   B115       -27545.3
144  ;
145
145     estimate;
146
146
146  ???????????????????????????????????????????????????????????????
146  ????????????     Now Fully General     ?????????????????????????
146  ???????????????????????????????????????????????????????????????
146   ?  Number of observations = 525      Log likelihood = -29346.7
146
146   ?  Number of observations = 525      Log likelihood = -29309.1
146   ?  Number of observations = 525      Log likelihood = -29308.9
146   ?   Below is new. Not yet converged.
146   ?  Number of observations = 525      Log likelihood = -29299.0
146   ?  Number of observations = 525      Log likelihood = -29299.0
146   ?  RC12        14.8286       60.2525        .246108        [.806]
146   ?  RC13        76143.1       17853.2        4.26496        [.000]
146   ?  RC22        350.532       54.6278        6.41673        [.000]
146   ?  RC23        77353.3       3645.79        21.2172        [.000]
146   ?  RC33        0.            0.             0.             [1.00]
146   ?  B330        -877141.      224389.        -3.90902       [.000]
146   ?  B331        -3989.56      837.078        -4.76605       [.000]
146   ?  B332        -367.030      56.7940        -6.46249       [.000]
146   ?  B333        8.64496       1.57290        5.49619        [.000]
146   ?  B334        24616.1       15050.2        1.63560        [.102]
```

```
|    146  ? B335      -.141242E+07  329418.       -4.28764      [.000]
|    146  ? B336      -3489.06      710.562       -4.91028      [.000]
|    146  ? B337      -98.3309      44.9359       -2.18825      [.029]
|    146  ? B338      2.95763       .923927       3.20115       [.001]
|    146  ? B339      -1470.74      7236.13       -.203249      [.839]
|    146  ? EW        -.020112      .438257E-02   -4.58902      [.000]
|    146  ? ER        -1.01533      .058140       -17.4634      [.000]
|    146  ? EA        -.158875E-05  .401148E-06   -3.96051      [.000]
|    146  ? ET        -.983078E-02  .327147E-02   -3.00500      [.003]
|    146  ? B220      100.242       467.996       .214193       [.830]
|    146  ? B221      -35.6121      1.99508       -17.8499      [.000]
|    146  ? B222      -1.91243      .160301       -11.9302      [.000]
|    146  ? B223      .033636       .544218E-02   6.18055       [.000]
|    146  ? B224      11.0151       11.5606       .952816       [.341]
|    146  ? B225      -1329.30      695.595       -1.91103      [.056]
|    146  ? B226      -17.1739      1.94800       -8.81617      [.000]
|    146  ? B227      -.605163      .121535       -4.97935      [.000]
|    146  ? B228      .304307E-02   .525858E-02   .578687       [.563]
|    146  ? B229      32.4560       15.6588       2.07270       [.038]
|    146  ? B110      -99979.3      20191.5       -4.95156      [.000]
|    146  ? B115      -140612.      29660.7       -4.74067      [.000]
|    146
|    146
|    146 set skip = 1;
|    147 const
|    147 fa11 1 fa22 0.01 fa33 17
|    147 fa12 0.1 fa13 0.1 fa23 0.1
|    147 fb12 10 fb13 10 fb23 10
|    147 fc11 11  fc22 0.01 fc33 0.02;
|    148 set eps=0;
|    149
|    149
|    149
|    149 Title ' **** General Model *****';
|    150
|    150  ?? Constants because of corner
|    150    const
|    150  RA22        0
|    150  RA13        0
|    150  RA12        0
|    150  RA11        0
|    150  RA23        0
|    150  RB23        -9.00000
|    150  RB13        -9.00000
|    150  RB12        -9.00000
|    150  ;
|    151
|    151   ?? Constants to aid estimation
|    151   const RC11       13034.9 ;
|    152   ? because rc's converge towards zero
|    152
|    152  param
|    152  RC12        14.8286
|    152  RC13        76143.1
|    152  RC22        350.532
|    152  RC23        77353.3
|    152  RC33        0.
|    152  B330        -877141.
|    152  B331        -3989.56
|    152  B332        -367.030
|    152  B333        8.64496
```

```
B334       24616.1
B335       -.141242E+07
B336       -3489.06
B337       -98.3309
B338       2.95763
B339       -1470.74
EW         -.020112
ER         -1.01533
EA         -.158875E-05
ET         -.983078E-02
B220       100.242
B221       -35.6121
B222       -1.91243
B223       .033636
B224       11.0151
B225       -1329.30
B226       -17.1739
B227       -.605163
B228       .304307E-02
B229       32.4560
B110       -99979.3
B115       -140612.
;

 estimate;




????????????????????????????????????????????????????????
?????????????   PROC ESTIMATE  ?????????????????????????
????????????????????????????????????????????????????????
Proc estimate;
set indic1=1; set indic2=2; set indic3=3;

? title 'SIX EQUATIONS IN FIML ';
if (branch=1); then; title " Naive Model -- Branch=1";
if (branch=2); then; title " Perfect Foresight Model -- Branch=2";
if (branch=3); then; title " Rational Expectations Model --
Branch=3";


if (skip=1); then; goto 600;
dot 1-3;
if (indic.=1); then; set eps=0.1;
if (indic.=2); then; set eps=0.01;
if (indic.=3); then; set eps=0;

fiml(endog=(ss2,ss3,ss4,ss5,ss6,ss7),
   maxit=20, maxsqz=12, tol=0.04, silent)
eqs2,eqs3,eqs4,eqs5,eqs6,eqs7;
enddot;


600 set eps=0;
fiml(endog=(ss1,ss3,ss4,ss5,ss6,ss7), maxit=30, maxsqz=16, tol=0.02)
eqs1,eqs3,eqs4,eqs5,eqs6,eqs7;

?***  verbose, maxit=10, maxsqz=15, tol=0.05
frml eqmut  mut =  ( - bb - (bb^2  -  4*aa*cc)^0.5)/2   ;
frml eqlamt lamt=  ( - bb + (bb^2  -  4*aa*cc)^0.5)/2   ;
```

```
186  frml eqbbt bbt=bb;
187  frml eqbb2t bb2t=bb^2;
188  frml eqrank1 rank1=  bb ;
189  frml eqrank2 rank2= (bb^2   -   4*aa*cc)^0.5    ;
190
190  eqsub eqlamt eqaa eqbb eqcc  eqra85-eqra89
190
190  eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
190  eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
190  eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
191  eqsub eqmut eqaa eqbb eqcc  eqra85-eqra89
191  eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
191  eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
191  eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
192  eqsub eqbbt eqbb eqra85-eqra89
192  eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
192  eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
192  eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
193  eqsub eqbb2t eqbb eqra85-eqra89
193  eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
193  eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
193  eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
194  eqsub eqrank1 eqaa eqbb eqcc  eqra85-eqra89
194  eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
194  eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
194  eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
195  eqsub eqrank2 eqaa eqbb eqcc  eqra85-eqra89
195  eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
195  eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
195  eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
196
196  eqsub eqaa eqra85-eqra89
196  eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
196  eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
196  eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
197  eqsub eqcc  eqra85-eqra89
197  eqop85-eqop89 eqow85-eqow89 eqor89 eqb185-eqb189 eqb285-eqb289 eqb389
197  eqa11-eqa13 eqa22-eqa23 eqa33 eqb12 eqb13 eqb23
197  eqc11 eqc22 eqc33 eqc12 eqc13 eqc23 ;
198
198  genr eqaa aaa; genr eqcc ccc; aaaccc= -4*aaa*ccc ;
201  genr eqbbt; genr eqbb2t;
203  genr eqlamt; genr eqmut; genr eqrank1; genr eqrank2;
207
207  genr eqs1 preds1; errs1=ss1-preds1; genr eqs2 preds2;
errs2=ss2-preds2;
211  genr eqs3 preds3; errs3=ss3-preds3; genr eqs5 preds5;
errs5=ss5-preds5;
215  genr eqs7 preds7; errs7=ss7-preds7;
217
217  If (@ifconv .ne. 1); then; title '******** NOT CONVERGED ********';
220  msd (terse, byvar) aaa ccc aaaccc bbt bb2t lamt mut rank1 rank2
220  preds1 errs1 preds2 errs2 preds3 errs3 preds5 errs5 preds7 errs7 ;
221
221  if excel=0; then; goto 800;    ? Skip excel file
224
224
224  write (file='output\singout.xls',format=excel) id85
224  hdwkhr85 nodep85 age85 hdwage85 hdwkhu85
224  hdwkhr86 nodep86 age86 hdwage86 hdwkhu86
224  hdwkhr87 nodep87 age87 hdwage87 hdwkhu87
```

```
224  hdwkhr88 nodep88 age88 hdwage88 hdwkhu88
224  hdwkhr89 nodep89 age89 sexhd hdwage89 capgain hdwkhu89
224  wun85 wun86 wun87 wun88 wun89
224  wth84 wth85 wth86 wth87 wth88 wth89 i85 i86 i87 i88 i89
224  y85 y86 y87 y88 y89 m endowm virtinc
224  s1 s2 s3 s4 s5 s6 s7 su3 su4 su5 su6 su7
224  ss1 ss2 ss3 ss4 ss5 ss6 ss7
224  rr86-rr89 stddevi reldevi stddevw reldevw iwth85-iwth89
224  aaa ccc aaaccc bbt bb2t
224  lamt mut rank1 rank2 preds1-preds3 preds5 preds7
224  errs1-errs3 errs5 errs7
224  ;
225
225
225    800  elast;
226
226  Endproc;
227
227  ??????????????? End of Estimate  ??????????????????????
227  ????????????????????????????????????????????????????????
227
227  ????????????????????????????????????????????????????????
227  ?????????????   PROC ELAST    ?????????????????????????
227  ????????????????????????????????????????????????????????
227  Proc elast;
228
228  frml eqesta esta = [   b330  +
228  (-a33/r - a13/(p+r) - a23/(w+r)  )*emu +
228  (c33*r  +  c13*p + c23*w)/emu  ];
229
229  frml eqestc estc = [   b110  +
229  (-a11/p - a12/(p+w) - a13/(p+r)  )*emu +
229  (c11*p  +   c12*w + c13*r)/emu ];
230
230  frml eqesth esth = [   b220   +
230  (-a22/w - a12/(p+w) - a23/(w+r)  )*emu +
230  (c22*w  +   c12*p + c23*r)/emu ];
231
231  frml eqemu emu=
231  { -[ Asset + b110*p + b220*w + b330*r] -
231     {  [ Asset + b110*p + b220*w + b330*r]^2 -
231      4*[-a11 -a12 -a13 -a22 -a23 -a33]*
231        [c11*p+2*c12*p*w+2*c13*p*r+c22*w^2+2*c23*w*r+c33*r^2] }^0.5
} /
231  2 / [-a11 -a12 -a13 -a22 -a23 -a33];
232
232  frml eqemup emup=   -b110/2/[-a11 -a12 -a13 -a22 -a23 -a33]  -
232  {     [  ( Asset + b110*p + b220*w + b330*r)^2 -
232
232      4*(-a11 -a12 -a13 -a22 -a23 -a33)*
232        (c11*p+2*c12*p*w+2*c13*p*r+c22*w^2+2*c23*w*r+c33*r^2)
]^(-0.5) *
232        [  ( Asset + b110*p + b220*w + b330*r)*b110 -
232      4*(-a11 -a12 -a13 -a22 -a23 -a33)*(c11*p+c12*w+c13*r) ]    } /
232  2 / [-a11 -a12 -a13 -a22 -a23 -a33];
233
233  frml eqemuw emuw=   -b220/2/[-a11 -a12 -a13 -a22 -a23 -a33]  -
233  {     [  ( Asset + b110*p + b220*w + b330*r)^2 -
233      4*(-a11 -a12 -a13 -a22 -a23 -a33)*
233        (c11*p+2*c12*p*w+2*c13*p*r+c22*w^2+2*c23*w*r+c33*r^2)
]^(-0.5) *
```

```
      [  ( Asset + b110*p + b220*w + b330*r)*b220 -
     4*(-a11 -a12 -a13 -a22 -a23 -a33)*(c12*p+c22*w+c23*r) ]    } /
2 / [-a11 -a12 -a13 -a22 -a23 -a33];

frml eqemur emur=   -b330/2/[-a11 -a12 -a13 -a22 -a23 -a33]  -
{     [  ( Asset + b110*p + b220*w + b330*r)^2 -
     4*(-a11 -a12 -a13 -a22 -a23 -a33)*
        (c11*p+2*c12*p*w+2*c13*p*r+c22*w^2+2*c23*w*r+c33*r^2)
]^(-0.5) *
      [  ( Asset + b110*p + b220*w + b330*r)*b330 -
     4*(-a11 -a12 -a13 -a22 -a23 -a33)*(c13*p+c23*w+c33*r) ]    } /
2 / [-a11 -a12 -a13 -a22 -a23 -a33];

frml eqemua emua=   -1/2/[-a11 -a12 -a13 -a22 -a23 -a33]  -
{     [  ( Asset + b110*p + b220*w + b330*r)^2 -
     4*(-a11 -a12 -a13 -a22 -a23 -a33)*
        (c11*p+2*c12*p*w+2*c13*p*r+c22*w^2+2*c23*w*r+c33*r^2)
]^(-0.5) *
      [  ( Asset + b110*p + b220*w + b330*r)  ]    } /
2 / [-a11 -a12 -a13 -a22 -a23 -a33];


frml eqelmup elmup = p*emup/emu;
frml eqelmuw elmuw = w*emuw/emu;
frml eqelmur elmur = r*emur/emu;
frml eqelmua elmua = asset*emua/emu;

????????????????????Frisch Elasticities
????????????
frml eqefcp efcp = p*((a11/p^2+a12/(p+w)^2+a13/(p+r)^2)*emu +
c11/emu )/estc;

frml eqefcw efcw = w*(a12/(p+w)^2*emu + c12/emu )/estc;

frml eqefcr efcr = r*(a13/(p+r)^2*emu + c13/emu )/estc;

frml eqefcm efcm = emu*(-a11/p - a12/(p+w) - a13/(p+r)
   - (c11*p + c12*w + c13*r)/emu^2)/estc;

????????????
frml eqefhp efhp = p*(a12/(p+w)^2*emu + c12/emu )/esth;

frml eqefhw efhw = w*((a22/w^2+a12/(p+w)^2+a23/(w+r)^2)*emu +
c22/emu )/esth;

frml eqefhr efhr = r*(a23/(w+r)^2*emu + c23/emu )/esth;

frml eqefhm efhm = emu*(-a22/w - a12/(p+w) - a23/(w+r)
   - (c12*p + c22*w + c23*r)/emu^2)/esth;

????????????
frml eqefap efap = p*(a13/(p+r)^2*emu + c13/emu )/esta;

frml eqefaw efaw = w*(a23/(w+r)^2*emu + c23/emu )/esta;

frml eqefar efar = r*((a33/r^2+a13/(p+r)^2+a23/(w+r)^2)*emu +
c33/emu )/esta;

frml eqefam efam = emu*(-a33/r - a13/(p+r) - a23/(w+r)
   - (c13*p + c23*w + c33*r)/emu^2)/esta;

```

```
252
252  ?????????????????
252  frml eqchecfc checfc = efcp + efcw + efcr + efcm;
253  frml eqchecfh checfh = efhp + efhw + efhr + efhm;
254  frml eqchecfa checfa = efap + efaw + efar + efam;
255  ? frml eqchecfm checfm = elmup + elmuw + elmur + elmua;
255
255  eqsub eqchecfc eqefcp eqefcw eqefcr eqefcm eqestc
255  eqemup eqemuw eqemur eqemua eqemu
255
255  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
256  eqsub eqchecfh eqefhp eqefhw eqefhr eqefhm eqesth
256  eqemup eqemuw eqemur eqemua eqemu
256  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
257  eqsub eqchecfa eqefap eqefaw eqefar eqefam eqesta
257  eqemup eqemuw eqemur eqemua eqemu
257  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
258
258  eqsub eqefcp eqestc eqemup eqemuw eqemur eqemua eqemu
258  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
259  eqsub eqefcw eqestc eqemup eqemuw eqemur eqemua eqemu
259  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
260  eqsub eqefcr eqestc eqemup eqemuw eqemur eqemua eqemu
260  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
261  eqsub eqefcm eqestc eqemup eqemuw eqemur eqemua eqemu
261  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
262
262  eqsub eqefhp eqesth eqemup eqemuw eqemur eqemua eqemu
262  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
263  eqsub eqefhw eqesth eqemup eqemuw eqemur eqemua eqemu
263  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
264  eqsub eqefhr eqesth eqemup eqemuw eqemur eqemua eqemu
264  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
265  eqsub eqefhm eqesth eqemup eqemuw eqemur eqemua eqemu
265  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
266
266  eqsub eqefap eqesta eqemup eqemuw eqemur eqemua eqemu
266  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
267  eqsub eqefaw eqesta eqemup eqemuw eqemur eqemua eqemu
267  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
268  eqsub eqefar eqesta eqemup eqemuw eqemur eqemua eqemu
268  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
269  eqsub eqefam eqesta eqemup eqemuw eqemur eqemua eqemu
269  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
270
270  ???????????????????Marshallian Elasticities
270  ????????????
270  frml eqelcp elcp = p*((a11/p^2+a12/(p+w)^2+a13/(p+r)^2)*emu + c11/emu
270  -(a11/p+a12/(p+w)+a13/(p+r))*emup - (c11*p + c12*w +
c13*r)/emu^2*emup)/estc;
271
271  frml eqelcw elcw = w*(a12/(p+w)^2*emu + c12/emu
271  -(a11/p+a12/(p+w)+a13/(p+r))*emuw - (c11*p + c12*w +
c13*r)/emu^2*emuw)/estc;
272
272  frml eqelcr elcr = r*(a13/(p+r)^2*emu + c13/emu
272  -(a11/p+a12/(p+w)+a13/(p+r))*emur - (c11*p + c12*w +
c13*r)/emu^2*emur)/estc;
273
273  frml eqelca elca = asset*(
273  -(a11/p+a12/(p+w)+a13/(p+r))*emua - (c11*p + c12*w +
```

```
c13*r)/emu^2*emua)/estc;

????????????
frml eqelhp elhp = p*(a12/(p+w)^2*emu + c12/emu
-(a22/w+a12/(p+w)+a23/(w+r))*emup - (c12*p + c22*w +
c23*r)/emu^2*emup)/esth;

frml eqelhw elhw = w*((a22/w^2+a12/(p+w)^2+a23/(w+r)^2)*emu + c22/emu
-(a22/w+a12/(p+w)+a23/(w+r))*emuw - (c12*p + c22*w +
c23*r)/emu^2*emuw)/esth;

frml eqelhr elhr = r*(a23/(w+r)^2*emu + c23/emu
-(a22/w+a12/(p+w)+a23/(w+r))*emur - (c12*p + c22*w +
c23*r)/emu^2*emur)/esth;

frml eqelha elha = asset*(
-(a22/w+a12/(p+w)+a23/(w+r))*emua - (c12*p + c22*w +
c23*r)/emu^2*emua)/esth;

????????????
frml eqelap elap = p*(a13/(p+r)^2*emu + c13/emu
-(a33/r+a13/(p+r)+a23/(w+r))*emup - (c13*p + c23*w +
c33*r)/emu^2*emup)/esta;

frml eqelaw elaw = w*(a23/(w+r)^2*emu + c23/emu
-(a33/r+a13/(p+r)+a23/(w+r))*emuw - (c13*p + c23*w +
c33*r)/emu^2*emuw)/esta;

frml eqelar elar = r*((a33/r^2+a13/(p+r)^2+a23/(w+r)^2)*emu + c33/emu
-(a33/r+a13/(p+r)+a23/(w+r))*emur - (c13*p + c23*w +
c33*r)/emu^2*emur)/esta;


frml eqelaa elaa = asset*(
-(a33/r+a13/(p+r)+a23/(w+r))*emua - (c13*p + c23*w +
c33*r)/emu^2*emua)/esta;


?????????????????
frml eqcheckc checkc = elcp + elcw + elcr + elca;
frml eqcheckh checkh = elhp + elhw + elhr + elha;
frml eqchecka checka = elap + elaw + elar + elaa;
frml eqcheckm checkm = elmup + elmuw + elmur + elmua;

eqsub eqcheckc eqelcp eqelcw eqelcr eqelca eqestc
eqemup eqemuw eqemur eqemua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
eqsub eqcheckh eqelhp eqelhw eqelhr eqelha eqesth
eqemup eqemuw eqemur eqemua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
eqsub eqchecka eqelap eqelaw eqelar eqelaa eqesta
eqemup eqemuw eqemur eqemua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
eqsub eqcheckm eqelmup eqelmuw eqelmur eqelmua
eqemup eqemuw eqemur eqemua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;


eqsub eqesta eqemu eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13
eqc22-eqc23 eqc33;
eqsub eqestc eqemu eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13
```

```
eqc22-eqc23 eqc33;
292  eqsub eqesth eqemu eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13
eqc22-eqc23 eqc33;
293  eqsub eqemu eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23
eqc33;
294
294  eqsub eqemup eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23
eqc33;
295
295  eqsub eqemuw eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23
eqc33;
296  eqsub eqemur eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23
eqc33;
297  eqsub eqemua eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23
eqc33;
298
298  eqsub eqelmup eqemup eqemu
298   eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
299  eqsub eqelmuw eqemuw eqemu
299   eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
300  eqsub eqelmur eqemur eqemu
300   eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
301  eqsub eqelmua eqemua eqemu
301   eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
302
302  eqsub eqelcp eqestc eqemup eqemuw eqemur eqemua eqemu
302  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
303  eqsub eqelcw eqestc eqemup eqemuw eqemur eqemua eqemu
303  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
304  eqsub eqelcr eqestc eqemup eqemuw eqemur eqemua eqemu
304  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
305  eqsub eqelca eqestc eqemup eqemuw eqemur eqemua eqemu
305  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
306
306  eqsub eqelhp eqesth eqemup eqemuw eqemur eqemua eqemu
306  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
307  eqsub eqelhw eqesth eqemup eqemuw eqemur eqemua eqemu
307  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
308  eqsub eqelhr eqesth eqemup eqemuw eqemur eqemua eqemu
308  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
309  eqsub eqelha eqesth eqemup eqemuw eqemur eqemua eqemu
309  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
310
310  eqsub eqelap eqesta eqemup eqemuw eqemur eqemua eqemu
310  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
311  eqsub eqelaw eqesta eqemup eqemuw eqemur eqemua eqemu
311  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
312  eqsub eqelar eqesta eqemup eqemuw eqemur eqemua eqemu
312  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
313  eqsub eqelaa eqesta eqemup eqemuw eqemur eqemua eqemu
313  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
314
314
314  ?? Long term elasticites apply to stochastic model only
314
314  ???????????????????Marshallian Elasticities - Long term
314  ????????????
314
314
314  frml eqelcpl elcpl = efcp + efcm*(elmup + ew*w+er*r+ea*asset);
315
```

```
frml eqelcwl elcwl = efcw + efcm*(elmuw - w*ew);

frml eqelcrl elcrl = efcr + efcm*(elmur - r*er);

frml eqelcal elcal =      + efcm*(elmua - asset*ea);

????????????

frml eqelhpl elhpl = efhp + efhm*(elmup + ew*w+er*r+ea*asset);

frml eqelhwl elhwl = efhw + efhm*(elmuw - w*ew);

frml eqelhrl elhrl = efhr + efhm*(elmur - r*er);

frml eqelhal elhal =      + efhm*(elmua - asset*ea);

????????????

frml eqelapl elapl = efap + efam*(elmup + ew*w+er*r+ea*asset);

frml eqelawl elawl = efaw + efam*(elmuw - w*ew);

frml eqelarl elarl = efar + efam*(elmur - r*er);

frml eqelaal elaal =      + efam*(elmua - asset*ea);


eqsub eqelcpl eqefcp eqefcm eqestc eqemup eqemuw eqemur eqemua
eqelmup eqelmuw eqelmur eqelmua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
eqsub eqelcwl eqefcw eqefcm eqestc eqemup eqemuw eqemur eqemua
eqelmup eqelmuw eqelmur eqelmua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
eqsub eqelcrl eqefcr eqefcm eqestc eqemup eqemuw eqemur eqemua
eqelmup eqelmuw eqelmur eqelmua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
eqsub eqelcal eqefcm eqestc eqemup eqemuw eqemur eqemua
eqelmup eqelmuw eqelmur eqelmua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;

eqsub eqelhpl eqefhp eqefhm eqesth eqemup eqemuw eqemur eqemua
eqelmup eqelmuw eqelmur eqelmua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
eqsub eqelhwl eqefhw eqefhm eqesth eqemup eqemuw eqemur eqemua
eqelmup eqelmuw eqelmur eqelmua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
eqsub eqelhrl eqefhr eqefhm eqesth eqemup eqemuw eqemur eqemua
eqelmup eqelmuw eqelmur eqelmua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
eqsub eqelhal eqefhm eqesth eqemup eqemuw eqemur eqemua
eqelmup eqelmuw eqelmur eqelmua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;

eqsub eqelapl eqefap eqefam eqesta eqemup eqemuw eqemur eqemua
eqelmup eqelmuw eqelmur eqelmua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
eqsub eqelawl eqefaw eqefam eqesta eqemup eqemuw eqemur eqemua
eqelmup eqelmuw eqelmur eqelmua eqemu
eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
```

```
336  eqsub eqelarl eqefar eqefam eqesta eqemup eqemuw eqemur eqemua
336  eqelmup eqelmuw eqelmur eqelmua eqemu
336  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
337  eqsub eqelaal eqefam eqesta eqemup eqemuw eqemur eqemua
337  eqelmup eqelmuw eqelmur eqelmua eqemu
337  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
338  ????????????????
338
338  frml eqchecgc checgc = elcpl + elcwl + elcrl + elcal;
339  frml eqchecgh checgh = elhpl + elhwl + elhrl + elhal;
340  frml eqchecga checga = elapl + elawl + elarl + elaal;
341
341  eqsub eqchecgc eqelcpl eqelcwl eqelcrl eqelcal eqefcp eqefcw eqefcr
341  eqefcm eqestc eqemup eqemuw eqemur eqemua eqelmup eqelmuw eqelmur
eqelmua
341  eqemu eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
342
342  eqsub eqchecgh eqelhpl eqelhwl eqelhrl eqelhal eqefhp eqefhw eqefhr
342  eqefhm eqesth eqemup eqemuw eqemur eqemua eqelmup eqelmuw eqelmur
eqelmua
342  eqemu eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
343
343  eqsub eqchecga eqelapl eqelawl eqelarl eqelaal eqefap eqefaw eqefar
343  eqefam eqesta eqemup eqemuw eqemur eqemua eqelmup eqelmuw eqelmur
eqelmua
343  eqemu eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
344
344
344  set p=1; set w=4.1; set r=0.93;
347  set asset=100000;
348
348  analyz eqcheckc eqcheckh eqchecka eqcheckm
348         eqchecfc eqchecfh eqchecfa
348         eqchecgc eqchecgh eqchecga;
349
349
349  title "p=1, w=4.1, r=0.93, asset=100000,";
350  set p=1; set w=4.1; set r=0.93; set asset=100000;
354
354  analyz
354  eqemu eqemup eqemuw eqemur eqemua
354  eqelmup eqelmuw eqelmur eqelmua
354  eqesta eqestc eqesth
354  eqefcp eqefcw eqefcr eqefcm
354  eqefhp eqefhw eqefhr eqefhm
354  eqefap eqefaw eqefar eqefam
354  eqelcp eqelcw eqelcr eqelca
354  eqelhp eqelhw eqelhr eqelha
354  eqelap eqelaw eqelar eqelaa
354
354  eqelcpl eqelcwl eqelcrl eqelcal
354  eqelhpl eqelhwl eqelhrl eqelhal
354  eqelapl eqelawl eqelarl eqelaal
354  ;
355
355  title "p=1, w=4.1, r=0.93, asset=50000,";
356  set p=1; set w=4.1; set r=0.93; set asset=50000;
360
360  analyz
360  eqemu eqemup eqemuw eqemur eqemua
360  eqelmup eqelmuw eqelmur eqelmua
```

```
360  eqesta eqestc eqesth
360  eqefcp eqefcw eqefcr eqefcm
360  eqefhp eqefhw eqefhr eqefhm
360  eqefap eqefaw eqefar eqefam
360  eqelcp eqelcw eqelcr eqelca
360  eqelhp eqelhw eqelhr eqelha
360  eqelap eqelaw eqelar eqelaa
360
360  eqelcpl eqelcwl eqelcrl eqelcal
360  eqelhpl eqelhwl eqelhrl eqelhal
360  eqelapl eqelawl eqelarl eqelaal
360  ;
361
361  ?? response; switched off because not helpful.
361
361
361  Endproc;
362  ??????????????? End of Elast  ??????????????????????
362  ????????????????????????????????????????????????????????
362
362  ?????????????????????????????????????????????????????????
362  ??????????? Response Module ?????????????????????????????
362  ?????????????????????????????????????????????????????????
362  proc response;
363
363
363
363  ????????????????????Frisch Responses
363  ????????????
363  frml eqrfcp rfcp = ((a11/p^2+a12/(p+w)^2+a13/(p+r)^2)*emu + c11/emu
);
364
364  frml eqrfcw rfcw = (a12/(p+w)^2*emu + c12/emu );
365
365  frml eqrfcr rfcr = (a13/(p+r)^2*emu + c13/emu );
366
366  frml eqrfcm rfcm = (-a11/p - a12/(p+w) - a13/(p+r)
366     - (c11*p + c12*w + c13*r)/emu^2);
367
367  ????????????
367  frml eqrfhp rfhp = (a12/(p+w)^2*emu + c12/emu );
368
368  frml eqrfhw rfhw = ((a22/w^2+a12/(p+w)^2+a23/(w+r)^2)*emu + c22/emu
);
369
369  frml eqrfhr rfhr = (a23/(w+r)^2*emu + c23/emu );
370
370  frml eqrfhm rfhm = (-a22/w - a12/(p+w) - a23/(w+r)
370     - (c12*p + c22*w + c23*r)/emu^2);
371
371  ????????????
371  frml eqrfap rfap = (a13/(p+r)^2*emu + c13/emu );
372
372  frml eqrfaw rfaw = (a23/(w+r)^2*emu + c23/emu );
373
373  frml eqrfar rfar = ((a33/r^2+a13/(p+r)^2+a23/(w+r)^2)*emu + c33/emu
);
374
374  frml eqrfam rfam = (-a33/r - a13/(p+r) - a23/(w+r)
374     - (c13*p + c23*w + c33*r)/emu^2);
375
```

```
375
375  ?????????????????
375  frml eqchecrc checrc = p*rfcp + w*rfcw + r*rfcr + emu*rfcm;
376  frml eqchecrh checrh = p*rfhp + w*rfhw + r*rfhr + emu*rfhm;
377  frml eqchecra checra = p*rfap + w*rfaw + r*rfar + emu*rfam;
378
378
378  eqsub eqchecrc eqrfcp eqrfcw eqrfcr eqrfcm eqemu
378  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
379  eqsub eqchecrh eqrfhp eqrfhw eqrfhr eqrfhm eqemu
379  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
380  eqsub eqchecra eqrfap eqrfaw eqrfar eqrfam eqemu
380  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
381
381  eqsub eqrfcp eqemu
381  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
382  eqsub eqrfcw eqemu
382  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
383  eqsub eqrfcr eqemu
383  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
384  eqsub eqrfcm eqemu
384  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
385
385  eqsub eqrfhp eqemu
385  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
386  eqsub eqrfhw eqemu
386  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
387  eqsub eqrfhr eqemu
387  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
388  eqsub eqrfhm eqemu
388  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
389
389  eqsub eqrfap eqemu
389  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
390  eqsub eqrfaw eqemu
390  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
391  eqsub eqrfar eqemu
391  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
392  eqsub eqrfam eqemu
392  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
393
393  ???????????????????Marshallian Responses
393  ????????????
393  frml eqrmcp rmcp = ((a11/p^2+a12/(p+w)^2+a13/(p+r)^2)*emu + c11/emu
393  -(a11/p+a12/(p+w)+a13/(p+r))*emup - (c11*p + c12*w +
c13*r)/emu^2*emup);
394
394  frml eqrmcw rmcw = (a12/(p+w)^2*emu + c12/emu
394  -(a11/p+a12/(p+w)+a13/(p+r))*emuw - (c11*p + c12*w +
c13*r)/emu^2*emuw);
395
395  frml eqrmcr rmcr = (a13/(p+r)^2*emu + c13/emu
395  -(a11/p+a12/(p+w)+a13/(p+r))*emur - (c11*p + c12*w +
c13*r)/emu^2*emur);
396
396  frml eqrmca rmca = (
396  -(a11/p+a12/(p+w)+a13/(p+r))*emua - (c11*p + c12*w +
c13*r)/emu^2*emua);
397
397  ????????????
397  frml eqrmhp rmhp = (a12/(p+w)^2*emu + c12/emu
```

```
397  -(a22/w+a12/(p+w)+a23/(w+r))*emup - (c12*p + c22*w +
c23*r)/emu^2*emup);
398
398  frml eqrmhw rmhw = ((a22/w^2+a12/(p+w)^2+a23/(w+r)^2)*emu + c22/emu
398  -(a22/w+a12/(p+w)+a23/(w+r))*emuw - (c12*p + c22*w +
c23*r)/emu^2*emuw);
399
399  frml eqrmhr rmhr = (a23/(w+r)^2*emu + c23/emu
399  -(a22/w+a12/(p+w)+a23/(w+r))*emur - (c12*p + c22*w +
c23*r)/emu^2*emur);
400
400  frml eqrmha rmha = (
400  -(a22/w+a12/(p+w)+a23/(w+r))*emua - (c12*p + c22*w +
c23*r)/emu^2*emua);
401
401  ????????????
401  frml eqrmap rmap = (a13/(p+r)^2*emu + c13/emu
401  -(a33/r+a13/(p+r)+a23/(w+r))*emup - (c13*p + c23*w +
c33*r)/emu^2*emup);
402
402  frml eqrmaw rmaw = (a23/(w+r)^2*emu + c23/emu
402  -(a33/r+a13/(p+r)+a23/(w+r))*emuw - (c13*p + c23*w +
c33*r)/emu^2*emuw);
403
403  frml eqrmar rmar = ((a33/r^2+a13/(p+r)^2+a23/(w+r)^2)*emu + c33/emu
403  -(a33/r+a13/(p+r)+a23/(w+r))*emur - (c13*p + c23*w +
c33*r)/emu^2*emur);
404
404  frml eqrmaa rmaa = (
404  -(a33/r+a13/(p+r)+a23/(w+r))*emua - (c13*p + c23*w +
c33*r)/emu^2*emua);
405
405
405
405  ????????????????????Marshallian Responses - Long term
405  ????????????
405  frml eqrlcp rlcp = ((a11/p^2+a12/(p+w)^2+a13/(p+r)^2)*emu + c11/emu
405  -(emup-emu*(-ew*w-er*r-ea*asset))*
405  ((a11/p+a12/(p+w)+a13/(p+r))+(c11*p + c12*w + c13*r)/emu^2));
406
406  frml eqrlcw rlcw = (a12/(p+w)^2*emu + c12/emu
406  -(emuw-emu*ew)*((a11/p+a12/(p+w)+a13/(p+r))+ (c11*p + c12*w +
c13*r)/emu^2));
407
407  frml eqrlcr rlcr = (a13/(p+r)^2*emu + c13/emu
407  -(emur-emu*er)*((a11/p+a12/(p+w)+a13/(p+r))+ (c11*p + c12*w +
c13*r)/emu^2));
408
408  frml eqrlca rlca = (
408  -(emua-emu*ea)*((a11/p+a12/(p+w)+a13/(p+r))+ (c11*p + c12*w +
c13*r)/emu^2));
409
409  ????????????
409  frml eqrlhp rlhp = (a12/(p+w)^2*emu + c12/emu
409  -(emup-emu*(-ew*w-er*r-ea*asset))*
409  ((a22/w+a12/(p+w)+a23/(w+r))+(c12*p + c22*w + c23*r)/emu^2));
410
410  frml eqrlhw rlhw = ((a22/w^2+a12/(p+w)^2+a23/(w+r)^2)*emu + c22/emu
410  -(emuw-emu*ew)*((a22/w+a12/(p+w)+a23/(w+r))+ (c12*p + c22*w +
c23*r)/emu^2));
411
```

```
411  frml eqrlhr rlhr = (a23/(w+r)^2*emu + c23/emu
411  -(emur-emu*er)*((a22/w+a12/(p+w)+a23/(w+r))+ (c12*p + c22*w +
c23*r)/emu^2));
412
412  frml eqrlha rlha = (
412  -(emua-emu*ea)*((a22/w+a12/(p+w)+a23/(w+r))+ (c12*p + c22*w +
c23*r)/emu^2));
413
413  ????????????
413  frml eqrlap rlap = (a13/(p+r)^2*emu + c13/emu
413  -(emup-emu*(-ew*w-er*r-ea*asset))*
413  ((a33/r+a13/(p+r)+a23/(w+r))+(c13*p + c23*w + c33*r)/emu^2));
414
414  frml eqrlaw rlaw = (a23/(w+r)^2*emu + c23/emu
414  -(emuw-emu*ew)*((a33/r+a13/(p+r)+a23/(w+r))+ (c13*p + c23*w +
c33*r)/emu^2));
415
415  frml eqrlar rlar = ((a33/r^2+a13/(p+r)^2+a23/(w+r)^2)*emu + c33/emu
415  -(emur-emu*er)*((a33/r+a13/(p+r)+a23/(w+r))+ (c13*p + c23*w +
c33*r)/emu^2));
416
416  frml eqrlaa rlaa = (
416  -(emua-emu*ea)*((a33/r+a13/(p+r)+a23/(w+r))+ (c13*p + c23*w +
c33*r)/emu^2));
417
417
417
417  eqsub eqrlcp eqemup eqemuw eqemur eqemua eqemu
417  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
418  eqsub eqrlcw eqemup eqemuw eqemur eqemua eqemu
418  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
419  eqsub eqrlcr eqemup eqemuw eqemur eqemua eqemu
419  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
420  eqsub eqrlca eqemup eqemuw eqemur eqemua eqemu
420  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
421
421  eqsub eqrlhp eqemup eqemuw eqemur eqemua eqemu
421  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
422  eqsub eqrlhw eqemup eqemuw eqemur eqemua eqemu
422  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
423  eqsub eqrlhr eqemup eqemuw eqemur eqemua eqemu
423  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
424  eqsub eqrlha eqemup eqemuw eqemur eqemua eqemu
424  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
425
425  eqsub eqrlap eqemup eqemuw eqemur eqemua eqemu
425  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
426  eqsub eqrlaw eqemup eqemuw eqemur eqemua eqemu
426  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
427  eqsub eqrlar eqemup eqemuw eqemur eqemua eqemu
427  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
428  eqsub eqrlaa eqemup eqemuw eqemur eqemua eqemu
428  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
429
429  ?????????????????
429  ?????????????????
429
429
429  frml eqchecsc checsc = p*rmcp + w*rmcw + r*rmcr + asset*rmca;
430  frml eqchecsh checsh = p*rmhp + w*rmhw + r*rmhr + asset*rmha;
431  frml eqchecsa checsa = p*rmap + w*rmaw + r*rmar + asset*rmaa;
```

```
432
432  eqsub eqchecsc eqrmcp eqrmcw eqrmcr eqrmca
432  eqemup eqemuw eqemur eqemua eqemu
432  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
433  eqsub eqchecsh eqrmhp eqrmhw eqrmhr eqrmha
433  eqemup eqemuw eqemur eqemua eqemu
433  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
434  eqsub eqchecsa eqrmap eqrmaw eqrmar eqrmaa
434  eqemup eqemuw eqemur eqemua eqemu
434  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
435
435  eqsub eqrmcp eqemup eqemuw eqemur eqemua eqemu
435  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
436  eqsub eqrmcw eqemup eqemuw eqemur eqemua eqemu
436  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
437  eqsub eqrmcr eqemup eqemuw eqemur eqemua eqemu
437  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
438  eqsub eqrmca eqemup eqemuw eqemur eqemua eqemu
438  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
439
439  eqsub eqrmhp eqemup eqemuw eqemur eqemua eqemu
439  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
440  eqsub eqrmhw eqemup eqemuw eqemur eqemua eqemu
440  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
441  eqsub eqrmhr eqemup eqemuw eqemur eqemua eqemu
441  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
442  eqsub eqrmha eqemup eqemuw eqemur eqemua eqemu
442  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
443
443  eqsub eqrmap eqemup eqemuw eqemur eqemua eqemu
443  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
444  eqsub eqrmaw eqemup eqemuw eqemur eqemua eqemu
444  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
445  eqsub eqrmar eqemup eqemuw eqemur eqemua eqemu
445  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
446  eqsub eqrmaa eqemup eqemuw eqemur eqemua eqemu
446  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
447
447  frml eqchecqc checqc = p*rlcp + w*rlcw + r*rlcr + asset*rlca;
448  frml eqchecqh checqh = p*rlhp + w*rlhw + r*rlhr + asset*rlha;
449  frml eqchecqa checqa = p*rlap + w*rlaw + r*rlar + asset*rlaa;
450
450  eqsub eqchecqc eqrlcp eqrlcw eqrlcr eqrlca eqemu
450  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
451  eqsub eqchecqh eqrlhp eqrlhw eqrlhr eqrlha eqemu
451  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
452  eqsub eqchecqa eqrlap eqrlaw eqrlar eqrlaa eqemu
452  eqa11-eqa13 eqa22-eqa23 eqa33 eqc11-eqc13 eqc22-eqc23 eqc33;
453
453
453  analyz eqchecrc eqchecrh eqchecra
453         eqchecsc eqchecsh eqchecsa
453         eqchecqc eqchecqh eqchecqa;
454
454
454  title "p=1, w=4.1, r=0.93, asset=100000,";
455  set p=1; set w=4.1; set r=0.93; set asset=100000;
459
459  analyz
459  eqrfcp eqrfcw eqrfcr eqrfcm
459  eqrfhp eqrfhw eqrfhr eqrfhm
```

```
|     459  eqrfap eqrfaw eqrfar eqrfam
|     459  eqrmcp eqrmcw eqrmcr eqrmca
|     459  eqrmhp eqrmhw eqrmhr eqrmha
|     459  eqrmap eqrmaw eqrmar eqrmaa
|     459  eqrlcp eqrlcw eqrlcr eqrlca
|     459  eqrlhp eqrlhw eqrlhr eqrlha
|     459  eqrlap eqrlaw eqrlar eqrlaa
|     459  ;
|     460
|     460  title "p=1, w=4.1, r=0.93, asset=50000,";
|     461  set p=1; set w=4.1; set r=0.93; set asset=50000;
|     465
|     465  analyz
|     465  eqrfcp eqrfcw eqrfcr eqrfcm
|     465  eqrfhp eqrfhw eqrfhr eqrfhm
|     465  eqrfap eqrfaw eqrfar eqrfam
|     465  eqrmcp eqrmcw eqrmcr eqrmca
|     465  eqrmhp eqrmhw eqrmhr eqrmha
|     465  eqrmap eqrmaw eqrmar eqrmaa
|     465  eqrlcp eqrlcw eqrlcr eqrlca
|     465  eqrlhp eqrlhw eqrlhr eqrlha
|     465  eqrlap eqrlaw eqrlar eqrlaa
|     465  ;
|     466
|     466
|     466  Endproc;
|     467  ??????????????? End of response ?????????????????????????
|     467  ?????????????????????????????????????????????????????????
|     467
|     467
|     467
|     467  999 stop;  end;
          EXECUTION
*******************************************************************************
|       0

Current sample:  1 to 525
```

Univariate statistics
=====================

Number of Observations: 525

	Mean	Std Dev	Minimum	Maximum
ID85	2680.08190	1883.17678	44.00000	7019.00000
HDWKHR85	970.56952	959.82351	0.00000	2976.00000
NODEP85	1.54095	1.05603	1.00000	9.00000
AGE85	57.96571	16.63652	21.00000	93.00000
HDWAGE85	4.60491	5.23612	0.00000	21.72000
HDWKHU85	1011.79619	980.98311	0.00000	2976.00000
HDWKHR86	945.11429	972.79327	0.00000	2880.00000
NODEP86	1.52190	1.07806	1.00000	9.00000
AGE86	58.88762	16.62579	22.00000	94.00000
HDWAGE86	4.72891	5.70656	0.00000	22.89000
HDWKHU86	985.65524	999.50521	0.00000	3522.00000
HDWKHR87	930.95619	972.68243	0.00000	2920.00000
NODEP87	1.44000	0.95045	1.00000	8.00000
AGE87	59.93333	16.60536	23.00000	95.00000
HDWAGE87	4.91549	6.01100	0.00000	26.32000
HDWKHU87	953.88952	983.74319	0.00000	3093.00000
HDWKHR88	904.08571	965.50253	0.00000	2968.00000

NODEP88	1.38286	0.90381	0.00000	8.00000
AGE88	60.92381	16.62215	24.00000	96.00000
HDWAGE88	4.99230	6.35733	0.00000	29.46000
HDWKHU88	926.13524	979.02009	0.00000	2992.00000
HDWKHR89	865.88381	965.03722	0.00000	2975.00000
NODEP89	1.33143	0.80116	0.00000	8.00000
AGE89	61.94286	16.61614	25.00000	97.00000
SEXHD	1.82476	0.38053	1.00000	2.00000
HDWAGE89	5.14116	6.73535	0.00000	31.30000
CAPGAIN	12714.82667	48378.39901	-167500.00000	450000.00000
HDWKHU89	885.78476	983.48884	0.00000	3014.00000
WUN85	3.61259	3.86858	0.00000	14.70600
WUN86	3.68422	4.19391	0.00000	17.17900
WUN87	3.79063	4.36381	0.00000	18.42400
WUN88	3.87624	4.64861	0.00000	19.20620
WUN89	4.09860	5.09794	0.00000	20.97100
WTH84	56938.42095	65903.65359	1100.00000	375000.00000
WTH85	59621.06300	68042.17992	1280.00000	465000.00000
WTH86	62303.70470	71670.12665	1260.00000	555000.00000
WTH87	64986.34677	76576.09478	1240.00000	645000.00000
WTH88	67668.98854	82532.48817	1220.00000	735000.00000
WTH89	70351.63048	89329.43139	1200.00000	825000.00000
I85	0.10784	0.13196	-0.19808	0.59587
I86	0.092410	0.10783	-0.16997	0.44869
I87	0.083189	0.097831	-0.16480	0.38772
I88	0.077536	0.088435	-0.18037	0.31343
I89	0.075428	0.091705	-0.19863	0.35829
Y85	3034.61143	6600.73701	-14833.00000	86535.00000
Y86	2943.94667	5371.07322	-12571.00000	22176.00000
Y87	3144.55810	5848.26318	-12641.00000	31060.00000
Y88	3472.44381	6342.48199	-16332.00000	31502.00000
Y89	4614.87810	14241.89949	-21928.00000	255952.00000
M	110154.78893	74988.42941	12131.26270	425861.09375
ENDOWM	80333.53499	72733.99423	4491.53467	425861.09375
VIRTINC	3261.45941	8206.42368	-28228.90820	47974.48438
S1	0.37086	0.21586	0.0088236	0.91486
S2	0.62914	0.21586	0.085143	0.99118
S3	0.061148	0.068533	0.00000	0.30359
S4	0.056318	0.063383	0.00000	0.23237
S5	0.053975	0.062160	0.00000	0.22002
S6	0.051166	0.063046	0.00000	0.24432
S7	0.051335	0.067456	0.00000	0.31237
SU3	0.064542	0.072744	0.00000	0.49639
SU4	0.058821	0.065131	0.00000	0.23237
SU5	0.055586	0.063414	0.00000	0.22002
SU6	0.052277	0.063779	0.00000	0.24432
SU7	0.052279	0.068305	0.00000	0.31237
SS1	45686.93238	52172.88113	869.90265	359401.59375
SS2	64467.85631	41382.87770	6397.20020	270458.15625
SS3	6293.67853	7457.80461	0.00000	27884.08008
SS4	6038.36227	7566.19704	0.00000	30772.19727
SS5	5912.18781	7804.29057	0.00000	39617.64063
SS6	5740.43923	8271.84886	0.00000	50062.32422
SS7	5836.58604	9075.19519	0.00000	66861.39063
RR86	0.91437	0.10004	0.62662	1.24700
RR87	0.85299	0.16891	0.44736	1.50236
RR88	0.80664	0.22843	0.32670	1.75355
RR89	0.76981	0.28402	0.26111	2.01450
AGEN85	12.96571	16.63652	-24.00000	48.00000
AGEN86	13.88762	16.62579	-23.00000	49.00000
AGEN87	14.93333	16.60536	-22.00000	50.00000

AGEN88	15.92381	16.62215	-21.00000	51.00000
AGEN89	16.94286	16.61614	-20.00000	52.00000
NODEP85N	0.20955	1.05603	-0.33140	7.66860
NODEP86N	0.19050	1.07806	-0.33140	7.66860
NODEP87N	0.10860	0.95045	-0.33140	6.66860
NODEP88N	0.051457	0.90381	-1.33140	6.66860
NODEP89N	0.000028569	0.80116	-1.33140	6.66860
STDDEVI	-0.22334	0.57022	-0.76790	2.72928
RELDEVI	-0.20987	0.63589	-0.85222	3.17460
STDDEVW	1.49057	2.65454	-0.59705	15.79130
RELDEVW	-0.047276	0.99782	-0.63495	3.37886
IWTH85	-3.56480D-06	0.000089053	-0.000058901	0.00072020
IWTH86	-8.17879D-06	0.000082349	-0.000059250	0.00073260
IWTH87	-9.72374D-06	0.000081340	-0.000059502	0.00074540
IWTH88	-8.81392D-06	0.000084571	-0.000059691	0.00075862
IWTH89	-9.37597D-07	0.00010869	-0.000059840	0.00077228

Rational Expectations Model Branch is 444
==

**** Basic Model *****
=======================

Full Information Maximum Likelihood
===================================

Equations: EQS1 EQS3 EQS4 EQS5 EQS6 EQS7

Endogenous variables: SS1 SS3 SS4 SS5 SS6 SS7

CONSTANTS:

	RC12	RC13	EPS	FC22	RC23
VALUE	0.00000	0.00000	0.00000	1.00000	0.00000

	FC11	FC33	RB23	FB23	RB13
VALUE	11.00000	1.00000	0.00000	0.00000	0.00000

	FB13	RA33	RA23	FA23	FA22
VALUE	0.00000	1.00000	0.00000	0.00000	0.00100000

	RA13	FA13	RA12	FA12	FA11
VALUE	0.00000	0.00000	0.00000	0.00000	1.00000

	B331	B332	B333	B334	B336
VALUE	0.00000	0.00000	0.00000	0.00000	0.00000

	B337	B338	B339	P89	P85
VALUE	0.00000	0.00000	0.00000	1.00000	0.87828

	P86	P87	P88	RR85	RB12
VALUE	0.90955	0.92646	0.96027	1.00000	0.00000

	FB12	B221	B222	B223	B224
VALUE	0.00000	0.00000	0.00000	0.00000	0.00000

	B226	B227	B228	B229	B111
VALUE	0.00000	0.00000	0.00000	0.00000	0.00000

	B112	B113	B114	B116	B117
VALUE	0.00000	0.00000	0.00000	0.00000	0.00000

	B118	B119	AA13	AA23	AA12
VALUE	0.00000	0.00000	1.00000	1.00000	1.00000

```
*** WARNING in command 183 Procedure FIML: Non-differentiable function
    (zero used) ====>     >=

*** WARNING in command 183 Procedure FIML: Non-differentiable function
    (zero used) ====>     >=

*** WARNING in command 183 Procedure FIML: Non-differentiable function
    (zero used) ====>     >=

*** WARNING in command 183 Procedure FIML: Non-differentiable function
    (zero used) ====>     >=

*** WARNING in command 183 Procedure FIML: Non-differentiable function
    (zero used) ====>     >=

*** NOTE:  LIMWARN limit reached.  Further warning messages will be
    suppressed.
NOTE => The model is linear in the variables.
Working space used: 447275
```

STARTING VALUES

	RC22	RC11	RC33	RA22	RA11
VALUE	2223.53000	28710.70000	29596.90000	0.0025734	0.00000

	B330	B335	EW	ER	EA
VALUE	285239.00000	-9581.63000	-0.17651	-0.62293	-1.02695D-06

	ET	B220	B225	B110	B115
VALUE	-0.013260	1713.87000	859.82900	-20239.70000	-30952.80000

```
F= 29610.748243  FNEW= 29610.748024  ISQZ=  1 STEP= 1.       CRIT= .76861E-03

CONVERGENCE ACHIEVED AFTER    1 ITERATIONS
```

2 FUNCTION EVALUATIONS.

Full Information Maximum Likelihood
==================================

Residual Covariance Matrix

	EQS1	EQS3	EQS4	EQS5
EQS1	8.03275D+08			
EQS3	-4.84750D+06	7958067.81858		
EQS4	-6.16098D+06	5883569.59395	7623944.96424	
EQS5	-8.34338D+06	4486705.70878	5109201.22994	6644469.82937
EQS6	-1.03705D+07	4401060.15731	4840643.97727	5305682.71883
EQS7	-4.30036D+06	3854925.45462	4504366.04341	4568382.18836

	EQS6	EQS7
EQS6	6673924.92450	
EQS7	5273681.04610	7035929.40279

Number of observations = 525 Log likelihood = -29610.7
Schwarz B.I.C. = 29671.2

Parameter	Estimate	Standard Error	t-statistic	P-value
RC22	2223.87	177.744	12.5116	[.000]
RC11	28692.8	4395.19	6.52823	[.000]
RC33	29564.2	5239.84	5.64220	[.000]
RA22	.257840E-02	.260874E-03	9.88368	[.000]
RA11	.264524E-04	.323880E-02	.816733E-02	[.993]
B330	285644.	35006.8	8.15967	[.000]
B335	-9552.53	15237.9	-.626895	[.531]
EW	-.176418	.576640E-02	-30.5942	[.000]
ER	-.622768	.220739	-2.82129	[.005]
EA	-.102789E-05	.727271E-06	-1.41335	[.158]
ET	-.013215	.659309E-02	-2.00434	[.045]
B220	1714.26	19.7885	86.6294	[.000]
B225	859.551	20.3015	42.3393	[.000]
B110	-20218.4	1973.20	-10.2465	[.000]
B115	-30935.3	2539.88	-12.1798	[.000]

Standard Errors computed from covariance of analytic first derivatives (BHHH)

Equation: EQS1
Dependent variable: SS1

Mean of dep. var.	= 45686.9	Std. error of regression	= 28342.1
Std. dev. of dep. var.	= 52172.9	R-squared	= .729420
Sum of squared residuals	= .421719E+12	Durbin-Watson	= 1.89962
Variance of residuals	= .803275E+09		

Equation: EQS3
Dependent variable: SS3

Mean of dep. var.	= 6293.68	Std. error of regression	= 2821.00
Std. dev. of dep. var.	= 7457.80	R-squared	= .871543
Sum of squared residuals	= .417799E+10	Durbin-Watson	= 1.83431

```
   Variance of residuals = .795807E+07

Equation: EQS4
Dependent variable: SS4

       Mean of dep. var. = 6038.36      Std. error of regression = 2761.15
  Std. dev. of dep. var. = 7566.20                     R-squared = .881399
Sum of squared residuals = .400257E+10             Durbin-Watson = 1.94518
   Variance of residuals = .762394E+07

Equation: EQS5
Dependent variable: SS5

       Mean of dep. var. = 5912.19      Std. error of regression = 2577.69
  Std. dev. of dep. var. = 7804.29                     R-squared = .908302
Sum of squared residuals = .348835E+10             Durbin-Watson = 1.99193
   Variance of residuals = .664447E+07

Equation: EQS6
Dependent variable: SS6

       Mean of dep. var. = 5740.44      Std. error of regression = 2583.39
  Std. dev. of dep. var. = 8271.85                     R-squared = .917388
Sum of squared residuals = .350381E+10             Durbin-Watson = 1.95231
   Variance of residuals = .667392E+07

Equation: EQS7
Dependent variable: SS7

       Mean of dep. var. = 5836.59      Std. error of regression = 2652.53
  Std. dev. of dep. var. = 9075.20                     R-squared = .922473
Sum of squared residuals = .369386E+10             Durbin-Watson = 1.81438
   Variance of residuals = .703593E+07

                              Univariate statistics
                              =====================

Number of Observations: 525
```

	Num.Obs	Mean	Std Dev	Minimum	Maximum
AAA	525.00000	-0.95038	0.53734	-6.79403	-0.11351
CCC	525.00000	4.22097D+09	1.54460D+09	1.89185D+09	1.32911D+10
AAACCC	525.00000	1.77159D+10	1.76039D+10	2.19909D+09	1.97498D+11
BBT	525.00000	14955.14469	145249.02225	-179920.96875	867230.68750
BB2T	525.00000	2.12807D+10	5.38410D+10	826.24011	7.52089D+11
LAMT	525.00000	78798.87108	48892.86390	3527.61670	232900.31250
MUT	525.00000	-93754.01595	113039.81761	-920849.00000	-9427.58789
RANK1	525.00000	14955.14469	145249.02225	-179920.96875	867230.68750
RANK2	525.00000	172552.88726	96123.59936	53687.92578	974467.37500
PREDS1	525.00000	46422.35996	52789.47255	-58010.60938	300949.34375
ERRS1	525.00000	-735.42753	28359.58616	-143019.21875	91559.39063
PREDS2	525.00000	59651.22374	26522.85164	584.15125	135910.98438
ERRS2	525.00000	4816.63264	31343.01323	-89338.10938	182922.90625
PREDS3	525.00000	5454.74204	6610.98027	-1259.76147	27912.53516
ERRS3	525.00000	838.93649	2695.94091	-10520.84473	13037.87109
PREDS5	525.00000	5060.16191	6850.71629	-1151.83252	39564.07422
ERRS5	525.00000	852.02590	2435.12148	-15642.41406	14631.17773
PREDS7	525.00000	5096.61401	8376.19194	-1104.62976	68254.14844
ERRS7	525.00000	739.97203	2549.65744	-18420.35938	14599.24023

Results of Parameter Analysis

Parameter	Estimate	Standard Error	t-statistic	P-value
CHECKC	.138778E-16	.181289E-17	7.65504	[.000]
CHECKH	-.832667E-16	.256814E-17	-32.4230	[.000]
CHECKA	.222045E-15	.246555E-16	9.00587	[.000]
CHECKM	1.	.307167E-16	.325556E+17	[.000]
CHECFC	.277556E-16	.131272E-16	2.11436	[.034]
CHECFH	-.277556E-16	.795968E-17	-3.48702	[.000]
CHECFA	-.888178E-15	.998532E-17	-88.9484	[.000]
CHECGC	.624500E-16	.776936E-17	8.03799	[.000]
CHECGH	0.	.455703E-17	0.	[1.00]
CHECGA	-.666134E-15	.136935E-15	-4.86461	[.000]

Wald Test for the Hypothesis that the given set of Parameters are jointly zero:

CHISQ(0) = 0.00000000 ; P-value = 1.00000

p=1, w=4.1, r=0.93, asset=100000,

Results of Parameter Analysis

Parameter	Estimate	Standard Error	t-statistic	P-value
EMU	356198.	31804.8	11.1995	[.000]
EMUP	-15354.6	1075.77	-14.2731	[.000]
EMUW	1799.85	18.4670	97.4627	[.000]
EMUR	285720.	34134.2	8.37050	[.000]
EMUA	.984536	.463944E-02	212.210	[.000]
ELMUP	-.043107	.402331E-02	-10.7143	[.000]
ELMUW	.020717	.172071E-02	12.0398	[.000]
ELMUR	.745989	.022609	32.9958	[.000]
ELMUA	.276401	.023620	11.7018	[.000]
ESTA	-95082.7	598.211	-158.945	[.000]
ESTC	-17916.5	540.543	-33.1454	[.000]
ESTH	1547.18	17.0132	90.9404	[.000]
EFCP	-.129529	.042487	-3.04871	[.002]
EFCW	0.	0.	0.	[1.00]
EFCR	0.	0.	0.	[1.00]
EFCM	.129529	.042487	3.04871	[.002]
EFHP	0.	0.	0.	[1.00]
EFHW	.181575	.013964	13.0034	[.000]
EFHR	0.	0.	0.	[1.00]
EFHM	-.181575	.013964	-13.0034	[.000]
EFAP	0.	0.	0.	[1.00]
EFAW	0.	0.	0.	[1.00]
EFAR	-4.05216	.355689	-11.3924	[.000]
EFAM	4.05216	.355689	11.3924	[.000]
ELCP	-.135113	.044130	-3.06173	[.002]
ELCW	.268346E-02	.985221E-03	2.72372	[.006]
ELCR	.096627	.030574	3.16042	[.002]
ELCA	.035802	.013033	2.74709	[.006]
ELHP	.782717E-02	.798992E-03	9.79631	[.000]

ELHW	.177814	.013770	12.9129	[.000]
ELHR	-.135453	.012381	-10.9408	[.000]
ELHA	-.050188	.469157E-02	-10.6974	[.000]
ELAP	-.174677	.012912	-13.5282	[.000]
ELAW	.083949	.898310E-03	93.4519	[.000]
ELAR	-1.02930	.596576E-02	-172.534	[.000]
ELAA	1.12002	.834344E-02	134.240	[.000]
ELCPL	-.317137	.104439	-3.03657	[.002]
ELCWL	.096374	.033671	2.86224	[.004]
ELCRL	.171647	.057438	2.98841	[.003]
ELCAL	.049116	.019360	2.53697	[.011]
ELHPL	.262991	.041099	6.39893	[.000]
ELHWL	.046478	.743172E-02	6.25394	[.000]
ELHRL	-.240617	.042885	-5.61076	[.000]
ELHAL	-.068852	.014706	-4.68174	[.000]
ELAPL	-5.86909	.909746	-6.45135	[.000]
ELAWL	3.01494	.233458	12.9143	[.000]
ELARL	1.31761	.851873	1.54673	[.122]
ELAAL	1.53654	.290593	5.28760	[.000]

Wald Test for the Hypothesis that the given set of Parameters are jointly zero:

CHISQ(10) = 28516411. ; P-value = 0.00000

p=1, w=4.1, r=0.93, asset=50000,

Results of Parameter Analysis

Parameter	Estimate	Standard Error	t-statistic	P-value
EMU	307073.	31587.1	9.72147	[.000]
EMUP	-14561.6	1000.69	-14.5516	[.000]
EMUW	1809.71	19.4280	93.1495	[.000]
EMUR	285168.	34040.0	8.37744	[.000]
EMUA	.980167	.576451E-02	170.035	[.000]
ELMUP	-.047421	.478699E-02	-9.90619	[.000]
ELMUW	.024163	.231959E-02	10.4169	[.000]
ELMUR	.863659	.014495	59.5842	[.000]
ELMUA	.159599	.015609	10.2251	[.000]
ESTA	-41894.6	646.784	-64.7737	[.000]
ESTC	-17545.5	581.621	-30.1665	[.000]
ESTH	1587.19	16.1278	98.4132	[.000]
EFCP	-.153269	.035235	-4.34996	[.000]
EFCW	0.	0.	0.	[1.00]
EFCR	0.	0.	0.	[1.00]
EFCM	.153269	.035235	4.34996	[.000]
EFHP	0.	0.	0.	[1.00]
EFHW	.163272	.012701	12.8549	[.000]
EFHR	0.	0.	0.	[1.00]
EFHM	-.163272	.012701	-12.8549	[.000]
EFAP	0.	0.	0.	[1.00]
EFAW	0.	0.	0.	[1.00]
EFAR	-7.94452	.825941	-9.61875	[.000]
EFAM	7.94452	.825941	9.61875	[.000]
ELCP	-.160537	.036693	-4.37519	[.000]
ELCW	.370345E-02	.103014E-02	3.59510	[.000]

ELCR	.132372	.029450	4.49474	[.000]
ELCA	.024461	.672673E-02	3.63646	[.000]
ELHP	.774252E-02	.758927E-03	10.2019	[.000]
ELHW	.159327	.012556	12.6896	[.000]
ELHR	-.141012	.012143	-11.6126	[.000]
ELHA	-.026058	.243384E-02	-10.7066	[.000]
ELAP	-.376736	.030406	-12.3900	[.000]
ELAW	.191964	.334912E-02	57.3177	[.000]
ELAR	-1.08316	.012275	-88.2407	[.000]
ELAA	1.26793	.021058	60.2123	[.000]
ELCPL	-.368045	.087760	-4.19379	[.000]
ELCWL	.114565	.028328	4.04424	[.000]
ELCRL	.221142	.056066	3.94430	[.000]
ELCAL	.032339	.986886E-02	3.27684	[.001]
ELHPL	.228794	.036199	6.32039	[.000]
ELHWL	.041230	.682817E-02	6.03822	[.000]
ELHRL	-.235575	.039323	-5.99071	[.000]
ELHAL	-.034449	.672018E-02	-5.12625	[.000]
ELAPL	-11.1327	1.85338	-6.00671	[.000]
ELAWL	5.93835	.540440	10.9880	[.000]
ELARL	3.51810	1.68917	2.08274	[.037]
ELAAL	1.67624	.286268	5.85548	[.000]

Wald Test for the Hypothesis that the given set of Parameters are jointly zero:

CHISQ(10) = 8768523.6 ; P-value = 0.00000

**** Demo Model *****
=====================

Full Information Maximum Likelihood
===================================

Equations: EQS1 EQS3 EQS4 EQS5 EQS6 EQS7

Endogenous variables: SS1 SS3 SS4 SS5 SS6 SS7

CONSTANTS:

	RC12	RC13	EPS	FC22	RC23
VALUE	0.00000	0.00000	0.00000	1.00000	0.00000

	FC11	FC33	RB23	FB23	RB13
VALUE	11.00000	1.00000	0.00000	0.00000	0.00000

	FB13	RA33	RA23	FA23	FA22
VALUE	0.00000	1.00000	0.00000	0.00000	0.00100000

	RA13	FA13	RA12	FA12	FA11
VALUE	0.00000	0.00000	0.00000	0.00000	1.00000

	P89	P85	P86	P87	P88
VALUE	1.00000	0.87828	0.90955	0.92646	0.96027

	RR85	RB12	FB12	B111	B112
VALUE	1.00000	0.00000	0.00000	0.00000	0.00000

	B113	B114	B116	B117	B118
VALUE	0.00000	0.00000	0.00000	0.00000	0.00000

	B119	AA13	AA23	AA12
VALUE	0.00000	1.00000	1.00000	1.00000

NOTE => The model is linear in the variables.
Working space used: 813963

STARTING VALUES

	RC22	RC11	RC33	RA22	RA11
VALUE	756.48100	9634.18000	21941.30000	0.0046042	0.033174

	B330	B331	B332	B333	B334
VALUE	-1183.62000	-4521.38000	-162.76700	4.62436	15040.80000

	B335	B336	B337	B338	B339
VALUE	-54760.20000	-2484.51000	-117.70700	2.90496	-6646.11000

	EW	ER	EA	ET	B220
VALUE	-0.22925	-0.34889	-0.000010596	-0.068935	1859.17000

	B221	B222	B223	B224	B225
VALUE	-34.77860	-1.44080	0.021480	-3.63520	1408.02000

	B226	B227	B228	B229	B110
VALUE	-20.32810	-0.85523	0.012911	39.58460	-23378.20000

	B115
VALUE	-25630.90000

F= 29363.595666 FNEW= 29363.595663 ISQZ= 1 STEP= 1. CRIT= .24932E-04

CONVERGENCE ACHIEVED AFTER 1 ITERATIONS

2 FUNCTION EVALUATIONS.

Full Information Maximum Likelihood
===================================

Residual Covariance Matrix

	EQS1	EQS3	EQS4	EQS5
EQS1	5.50979D+08			
EQS3	-1.33771D+06	7018771.00693		
EQS4	-1.21669D+06	4981131.77258	6612606.75928	

EQS5	-4.00808D+06	3601074.65106	4335443.61357	5545054.87824
EQS6	-4.21776D+06	3511098.77443	4118436.35703	4249043.79387
EQS7	1282378.90490	3095837.36035	3861321.64847	3804228.17598

	EQS6	EQS7
EQS6	5605642.86227	
EQS7	4360729.01949	5872686.84620

Number of observations = 525 Log likelihood = -29363.6
Schwarz B.I.C. = 29488.5

Parameter	Estimate	Standard Error	t-statistic	P-value
RC22	756.763	82.8449	9.13470	[.000]
RC11	9637.10	1135.53	8.48684	[.000]
RC33	21946.8	2215.93	9.90408	[.000]
RA22	.460377E-02	.563840E-03	8.16502	[.000]
RA11	.033180	.592833E-02	5.59686	[.000]
B330	-1165.40	17670.4	-.065952	[.947]
B331	-4522.81	776.534	-5.82436	[.000]
B332	-162.618	53.8203	-3.02149	[.003]
B333	4.62172	1.55768	2.96705	[.003]
B334	15037.4	18890.5	.796030	[.426]
B335	-54748.2	6840.44	-8.00361	[.000]
B336	-2484.33	444.885	-5.58420	[.000]
B337	-117.697	30.8212	-3.81869	[.000]
B338	2.90471	.689035	4.21562	[.000]
B339	-6645.33	4575.37	-1.45241	[.146]
EW	-.229225	.010115	-22.6612	[.000]
ER	-.348836	.479909	-.726879	[.467]
EA	-.105942E-04	.173142E-05	-6.11883	[.000]
ET	-.068917	.015720	-4.38392	[.000]
B220	1859.11	34.4243	54.0059	[.000]
B221	-34.7792	1.99060	-17.4718	[.000]
B222	-1.44044	.136691	-10.5379	[.000]
B223	.021471	.544779E-02	3.94120	[.000]
B224	-3.63477	12.9627	-.280402	[.779]
B225	1407.97	30.6290	45.9685	[.000]
B226	-20.3276	2.06151	-9.86050	[.000]
B227	-.855218	.123273	-6.93761	[.000]
B228	.012911	.556966E-02	2.31808	[.020]
B229	39.5843	15.3562	2.57774	[.010]
B110	-23375.8	1209.77	-19.3224	[.000]
B115	-25630.4	1125.53	-22.7719	[.000]

Standard Errors computed from covariance of analytic first derivatives (BHHH)

Equation: EQS1
Dependent variable: SS1

Mean of dep. var. = 45686.9 Std. error of regression = 23472.9
Std. dev. of dep. var. = 52172.9 R-squared = .799448
Sum of squared residuals = .289264E+12 Durbin-Watson = 1.85097
Variance of residuals = .550979E+09

Equation: EQS3
Dependent variable: SS3

Mean of dep. var. = 6293.68 Std. error of regression = 2649.30

```
  Std. dev. of dep. var. = 7457.80                        R-squared = .879847
Sum of squared residuals = .368485E+10                Durbin-Watson = 1.88397
   Variance of residuals = .701877E+07

Equation: EQS4
Dependent variable: SS4

       Mean of dep. var. = 6038.36      Std. error of regression = 2571.50
  Std. dev. of dep. var. = 7566.20                        R-squared = .890738
Sum of squared residuals = .347162E+10                Durbin-Watson = 1.92089
   Variance of residuals = .661261E+07

Equation: EQS5
Dependent variable: SS5

       Mean of dep. var. = 5912.19      Std. error of regression = 2354.79
  Std. dev. of dep. var. = 7804.29                        R-squared = .917147
Sum of squared residuals = .291115E+10                Durbin-Watson = 2.03292
   Variance of residuals = .554505E+07

Equation: EQS6
Dependent variable: SS6

       Mean of dep. var. = 5740.44      Std. error of regression = 2367.62
  Std. dev. of dep. var. = 8271.85                        R-squared = .923949
Sum of squared residuals = .294296E+10                Durbin-Watson = 1.97372
   Variance of residuals = .560564E+07

Equation: EQS7
Dependent variable: SS7

       Mean of dep. var. = 5836.59      Std. error of regression = 2423.36
  Std. dev. of dep. var. = 9075.20                        R-squared = .931474
Sum of squared residuals = .308316E+10                Durbin-Watson = 1.82283
   Variance of residuals = .587269E+07
```

Univariate statistics
=====================

Number of Observations: 525

	Num.Obs	Mean	Std Dev	Minimum	Maximum
AAA	525.00000	-1.21086	1.09106	-21.73155	-0.32045
CCC	525.00000	7.64121D+08	4.38409D+08	2.45200D+08	4.19478D+09
AAACCC	525.00000	3.90965D+09	4.05836D+09	4.55951D+08	4.79652D+10
BBT	525.00000	-80686.60840	95951.29954	-436525.65625	253271.46875
BB2T	525.00000	1.56994D+10	2.16687D+10	889334.62500	1.90555D+11
LAMT	525.00000	102354.94846	72537.38096	1797.46631	444184.03125
MUT	525.00000	-21668.34012	38201.40278	-257908.35938	-1560.75720
RANK1	525.00000	-80686.60840	95951.29954	-436525.65625	253271.46875
RANK2	525.00000	124023.28891	65079.84234	26518.98828	451842.43750
PREDS1	525.00000	46355.24464	49031.44946	-61451.56250	286264.12500
ERRS1	525.00000	-668.31225	23485.80370	-125939.94531	92446.60156
PREDS2	525.00000	60998.92042	32415.51383	3984.67383	166950.87500
ERRS2	525.00000	3468.93590	25878.72214	-82231.62500	163627.56250
PREDS3	525.00000	5710.47258	6902.51037	-2475.31421	28997.58789
ERRS3	525.00000	583.20595	2586.77159	-10505.50684	11651.27344
PREDS5	525.00000	5294.86423	7116.92777	-992.62952	40273.46094
ERRS5	525.00000	617.32358	2274.60354	-13738.66016	14372.86719
PREDS7	525.00000	5363.41393	8632.96510	-971.12183	66381.87500
ERRS7	525.00000	473.17211	2378.98618	-16042.53906	15214.80469

Results of Parameter Analysis

Parameter	Estimate	Standard Error	t-statistic	P-value
CHECKC	-.277556E-16	.143021E-16	-1.94066	[.052]
CHECKH	-.138778E-16	.559024E-17	-2.48250	[.013]
CHECKA	0.	.833997E-16	0.	[1.00]
CHECKM	1.	.603034E-16	.165828E+17	[.000]
CHECFC	0.	.919198E-18	0.	[1.00]
CHECFH	-.138778E-16	.210000E-17	-6.60847	[.000]
CHECFA	0.	.218475E-17	0.	[1.00]
CHECGC	-.111022E-15	.977661E-17	-11.3559	[.000]
CHECGH	.277556E-16	.241497E-17	11.4931	[.000]
CHECGA	-.444089E-15	.688052E-17	-64.5430	[.000]

Wald Test for the Hypothesis that the given set of Parameters are jointly zero:

CHISQ(0) = 0.00000000 ; P-value = 1.00000

p=1, w=4.1, r=0.93, asset=100000,

Results of Parameter Analysis

Parameter	Estimate	Standard Error	t-statistic	P-value
EMU	85954.3	15668.6	5.48576	[.000]
EMUP	-19146.1	902.272	-21.2199	[.000]
EMUW	1727.14	53.2587	32.4293	[.000]
EMUR	8354.79	14860.9	.562198	[.574]
EMUA	.902492	.020732	43.5303	[.000]
ELMUP	-.222748	.041937	-5.31147	[.000]
ELMUW	.082384	.013029	6.32335	[.000]
ELMUR	.090396	.144370	.626145	[.531]
ELMUA	1.04997	.172008	6.10416	[.000]
ESTA	-88377.9	877.854	-100.675	[.000]
ESTC	-25147.2	810.475	-31.0277	[.000]
ESTH	1789.92	25.9852	68.8822	[.000]
EFCP	-.156378	.026209	-5.96650	[.000]
EFCW	0.	0.	0.	[1.00]
EFCR	0.	0.	0.	[1.00]
EFCM	.156378	.026209	5.96650	[.000]
EFHP	0.	0.	0.	[1.00]
EFHW	.069184	.011013	6.28184	[.000]
EFHR	0.	0.	0.	[1.00]
EFHM	-.069184	.011013	-6.28184	[.000]
EFAP	0.	0.	0.	[1.00]
EFAW	0.	0.	0.	[1.00]
EFAR	-1.10475	.184580	-5.98521	[.000]
EFAM	1.10475	.184580	5.98521	[.000]
ELCP	-.191210	.027015	-7.07798	[.000]
ELCW	.012883	.158107E-02	8.14834	[.000]
ELCR	.014136	.024295	.581859	[.561]

ELCA	.164191	.019971	8.22157	[.000]
ELHP	.015410	.183490E-02	8.39852	[.000]
ELHW	.063484	.010804	5.87596	[.000]
ELHR	-.625394E-02	.010767	-.580842	[.561]
ELHA	-.072640	.791432E-02	-9.17835	[.000]
ELAP	-.246080	.013421	-18.3360	[.000]
ELAW	.091014	.173113E-02	52.5749	[.000]
ELAR	-1.00488	.841904E-02	-119.358	[.000]
ELAA	1.15995	.012722	91.1736	[.000]
ELCPL	-.554579	.115362	-4.80728	[.000]
ELCWL	.159850	.025452	6.28042	[.000]
ELCRL	.064868	.079320	.817792	[.413]
ELCAL	.329862	.045674	7.22213	[.000]
ELHPL	.176170	.039636	4.44465	[.000]
ELHWL	-.153627E-02	.298821E-02	-.514108	[.607]
ELHRL	-.028698	.034811	-.824394	[.410]
ELHAL	-.145935	.017500	-8.33928	[.000]
ELAPL	-2.81314	.690166	-4.07604	[.000]
ELAWL	1.12928	.175583	6.43162	[.000]
ELARL	-.646484	.501286	-1.28965	[.197]
ELAAL	2.33035	.241419	9.65272	[.000]

Wald Test for the Hypothesis that the given set of Parameters are jointly zero:

CHISQ(9) = 12646273. ; P-value = 0.00000

p=1, w=4.1, r=0.93, asset=50000,

Results of Parameter Analysis

Parameter	Estimate	Standard Error	t-statistic	P-value
EMU	43463.6	13515.3	3.21589	[.001]
EMUP	-14553.3	2316.36	-6.28281	[.000]
EMUW	1498.71	177.996	8.41991	[.000]
EMUR	14816.0	11217.1	1.32084	[.187]
EMUA	.761866	.093742	8.12727	[.000]
ELMUP	-.334838	.064325	-5.20542	[.000]
ELMUW	.141376	.028518	4.95737	[.000]
ELMUR	.317021	.142196	2.22946	[.026]
ELMUA	.876441	.174202	5.03118	[.000]
ESTA	-37594.3	979.128	-38.3957	[.000]
ESTC	-22681.1	890.060	-25.4826	[.000]
ESTH	1864.33	27.7772	67.1175	[.000]
EFCP	-.157794	.022153	-7.12298	[.000]
EFCW	0.	0.	0.	[1.00]
EFCR	0.	0.	0.	[1.00]
EFCM	.157794	.022153	7.12298	[.000]
EFHP	0.	0.	0.	[1.00]
EFHW	.055155	.683994E-02	8.06364	[.000]
EFHR	0.	0.	0.	[1.00]
EFHM	-.055155	.683994E-02	-8.06364	[.000]
EFAP	0.	0.	0.	[1.00]
EFAW	0.	0.	0.	[1.00]
EFAR	-1.51729	.340809	-4.45201	[.000]
EFAM	1.51729	.340809	4.45201	[.000]

ELCP	-.210629	.029057	-7.24885	[.000]
ELCW	.022308	.557960E-02	3.99819	[.000]
ELCR	.050024	.022715	2.20228	[.028]
ELCA	.138297	.033768	4.09550	[.000]
ELHP	.018468	.293742E-02	6.28712	[.000]
ELHW	.047357	.668325E-02	7.08596	[.000]
ELHR	-.017485	.888223E-02	-1.96856	[.049]
ELHA	-.048340	.877265E-02	-5.51030	[.000]
ELAP	-.508044	.046028	-11.0378	[.000]
ELAW	.214508	.747808E-02	28.6849	[.000]
ELAR	-1.03627	.017537	-59.0917	[.000]
ELAA	1.32981	.048933	27.1761	[.000]
ELCPL	-.493704	.085491	-5.77492	[.000]
ELCWL	.170606	.022673	7.52457	[.000]
ELCRL	.101215	.074070	1.36649	[.172]
ELCAL	.221882	.044274	5.01152	[.000]
ELHPL	.117413	.025737	4.56197	[.000]
ELHWL	-.447843E-02	.269407E-02	-1.66233	[.096]
ELHRL	-.035378	.026917	-1.31438	[.189]
ELHAL	-.077556	.010942	-7.08763	[.000]
ELAPL	-3.22998	.959460	-3.36646	[.001]
ELAWL	1.64049	.319893	5.12823	[.000]
ELARL	-.544040	.692203	-.785955	[.432]
ELAAL	2.13354	.225650	9.45506	[.000]

Wald Test for the Hypothesis that the given set of Parameters are jointly zero:

CHISQ(10) = 0.78303706E+18 ; P-value = 0.00000

**** T/S Model *****
====================

Full Information Maximum Likelihood
===================================

Equations: EQS1 EQS3 EQS4 EQS5 EQS6 EQS7

Endogenous variables: SS1 SS3 SS4 SS5 SS6 SS7

CONSTANTS:

	RC13	EPS	FC22	RC23	FC11
VALUE	0.00000	0.00000	1.00000	0.00000	11.00000

	FC33	RB23	FB23	RB13	FB13
VALUE	1.00000	0.00000	0.00000	0.00000	0.00000

	RA33	RA23	FA23	RA22	FA22
VALUE	1.00000	0.00000	0.00000	0.00000	0.00100000

	RA13	FA13	FA12	FA11	P89
VALUE	0.00000	0.00000	0.00100000	1.00000	1.00000

	P85	P86	P87	P88	RR85
VALUE	0.87828	0.90955	0.92646	0.96027	1.00000

	FB12	B111	B112	B113	B114
VALUE	0.00100000	0.00000	0.00000	0.00000	0.00000

	B116	B117	B118	B119	AA13
VALUE	0.00000	0.00000	0.00000	0.00000	1.00000

	AA23	AA12
VALUE	1.00000	1.00000

NOTE => The model is linear in the variables.
Working space used: 874039

STARTING VALUES

	RC12	RC22	RC11	RC33	RA12
VALUE	247.21500	723.52300	13034.90000	29836.00000	0.0056351

	RA11	B330	B331	B332	B333
VALUE	0.032811	7277.14000	-4354.15000	-191.48500	5.07762

	B334	B335	B336	B337	B338
VALUE	15779.90000	-59582.70000	-2741.34000	-115.14500	2.90590

	B339	EW	ER	EA	ET
VALUE	-5367.69000	-0.19492	-0.46521	-0.000010224	-0.076539

	RB12	B220	B221	B222	B223
VALUE	-266.20600	1856.04000	-33.78900	-1.46942	0.020672

	B224	B225	B226	B227	B228
VALUE	-0.85084	1384.76000	-20.41620	-0.82347	0.010089

	B229	B110	B115
VALUE	38.57970	-24126.50000	-27545.30000

F= 29354.631064 FNEW= 29354.631022 ISQZ= 1 STEP= 1. CRIT= .70076E-03

CONVERGENCE ACHIEVED AFTER 1 ITERATIONS

2 FUNCTION EVALUATIONS.

Full Information Maximum Likelihood

Residual Covariance Matrix

	EQS1	EQS3	EQS4	EQS5
EQS1	5.57575D+08			

EQS3	-2.97034D+06	6876181.49740			
EQS4	-2.72326D+06	4851492.72491	6499135.89072		
EQS5	-5.67246D+06	3489676.51832	4253960.65567	5476574.59446	
EQS6	-6.16917D+06	3366133.64501	3993100.44057	4129502.20383	
EQS7	-833959.85238	2944039.47284	3738843.94299	3696829.91314	

	EQS6	EQS7
EQS6	5428964.10440	
EQS7	4195866.92327	5712406.53893

Number of observations = 525 Log likelihood = -29354.6
Schwarz B.I.C. = 29483.5

Parameter	Estimate	Standard Error	t-statistic	P-value
RC12	245.936	67.5396	3.64136	[.000]
RC22	722.453	84.1559	8.58469	[.000]
RC11	13006.1	1783.50	7.29249	[.000]
RC33	29773.1	3634.84	8.19104	[.000]
RA12	.562413E-02	.699337E-03	8.04209	[.000]
RA11	.032804	.669559E-02	4.89928	[.000]
B330	7239.08	18687.1	.387385	[.698]
B331	-4351.38	852.572	-5.10383	[.000]
B332	-192.060	56.7270	-3.38568	[.001]
B333	5.08941	1.65942	3.06698	[.002]
B334	15810.5	15630.6	1.01151	[.312]
B335	-59631.4	8248.83	-7.22907	[.000]
B336	-2741.18	482.715	-5.67867	[.000]
B337	-115.153	32.4794	-3.54540	[.000]
B338	2.90620	.720474	4.03374	[.000]
B339	-5360.08	4905.41	-1.09269	[.275]
EW	-.195287	.011549	-16.9099	[.000]
ER	-.465873	.420700	-1.10738	[.268]
EA	-.102359E-04	.148881E-05	-6.87521	[.000]
ET	-.076702	.015340	-5.00027	[.000]
RB12	-266.206	0.	0.	[1.00]
B220	1856.47	36.2370	51.2312	[.000]
B221	-33.7884	1.97332	-17.1226	[.000]
B222	-1.47057	.137078	-10.7279	[.000]
B223	.020706	.525602E-02	3.93943	[.000]
B224	-.858932	12.0349	-.071370	[.943]
B225	1385.28	32.9326	42.0641	[.000]
B226	-20.4163	1.98797	-10.2699	[.000]
B227	-.823363	.121752	-6.76264	[.000]
B228	.010088	.538871E-02	1.87207	[.061]
B229	38.6231	15.4754	2.49577	[.013]
B110	-24125.9	1467.96	-16.4350	[.000]
B115	-27538.2	1416.50	-19.4410	[.000]

Standard Errors computed from covariance of analytic first derivatives (BHHH)

Equation: EQS1
Dependent variable: SS1

Mean of dep. var. = 45686.9 Std. error of regression = 23613.0
Std. dev. of dep. var. = 52172.9 R-squared = .796630
Sum of squared residuals = .292727E+12 Durbin-Watson = 1.85271
Variance of residuals = .557575E+09

```
Equation: EQS3
Dependent variable: SS3

       Mean of dep. var. = 6293.68      Std. error of regression = 2622.25
   Std. dev. of dep. var. = 7457.80                    R-squared = .881382
Sum of squared residuals = .361000E+10             Durbin-Watson = 1.93032
   Variance of residuals = .687618E+07

Equation: EQS4
Dependent variable: SS4

       Mean of dep. var. = 6038.36      Std. error of regression = 2549.34
   Std. dev. of dep. var. = 7566.20                    R-squared = .891827
Sum of squared residuals = .341205E+10             Durbin-Watson = 1.94862
   Variance of residuals = .649914E+07

Equation: EQS5
Dependent variable: SS5

       Mean of dep. var. = 5912.19      Std. error of regression = 2340.21
   Std. dev. of dep. var. = 7804.29                    R-squared = .917879
Sum of squared residuals = .287520E+10             Durbin-Watson = 2.07344
   Variance of residuals = .547657E+07

Equation: EQS6
Dependent variable: SS6

       Mean of dep. var. = 5740.44      Std. error of regression = 2330.01
   Std. dev. of dep. var. = 8271.85                    R-squared = .926258
Sum of squared residuals = .285021E+10             Durbin-Watson = 1.99832
   Variance of residuals = .542896E+07

Equation: EQS7
Dependent variable: SS7

       Mean of dep. var. = 5836.59      Std. error of regression = 2390.06
   Std. dev. of dep. var. = 9075.20                    R-squared = .932796
Sum of squared residuals = .299901E+10             Durbin-Watson = 1.85929
   Variance of residuals = .571241E+07
```

Univariate statistics
=====================

Number of Observations: 525

	Num.Obs	Mean	Std Dev	Minimum	Maximum
AAA	525.00000	-1.17678	0.76162	-13.57284	-0.34645
CCC	525.00000	1.38124D+09	7.10418D+08	4.41243D+08	6.45830D+09
AAACCC	525.00000	6.95893D+09	6.40448D+09	8.31058D+08	5.99858D+10
BBT	525.00000	-92559.94357	100137.85670	-471587.43750	255624.70313
BB2T	525.00000	1.85758D+10	2.52493D+10	213321.29688	2.22395D+11
LAMT	525.00000	118291.66808	77408.38569	3391.56226	484495.46875
MUT	525.00000	-25731.72463	37713.68710	-263734.15625	-2585.28857
RANK1	525.00000	-92559.94357	100137.85670	-471587.43750	255624.70313
RANK2	525.00000	144023.39298	69290.47953	36036.66016	497403.53125
PREDS1	525.00000	46684.63998	48583.13848	-59078.38281	281020.09375
ERRS1	525.00000	-997.70758	23614.43455	-117607.14844	90073.42188
PREDS2	525.00000	61064.66442	32586.76579	3943.62476	165378.84375
ERRS2	525.00000	3403.19195	26293.73414	-84418.17188	156490.87500
PREDS3	525.00000	5787.51067	6813.82067	-1861.31873	28721.88086
ERRS3	525.00000	506.16786	2575.38524	-10524.31738	11545.11035

PREDS5	525.00000	5367.64683	7043.17265	-771.85956	39742.09375
ERRS5	525.00000	544.54098	2278.14296	-12113.86719	14269.29883
PREDS7	525.00000	5445.76415	8576.69976	-15.35573	66163.33594
ERRS7	525.00000	390.82189	2360.14297	-14266.80078	14793.35547

Results of Parameter Analysis

Parameter	Estimate	Standard Error	t-statistic	P-value
CHECKC	-.277556E-16	.101019E-16	-2.74757	[.006]
CHECKH	.277556E-16	.558614E-17	4.96865	[.000]
CHECKA	-.222045E-15	.326854E-16	-6.79338	[.000]
CHECKM	1.000000	.143740E-16	.695700E+17	[.000]
CHECFC	.277556E-16	.508013E-17	5.46356	[.000]
CHECFH	-.138778E-16	.255334E-17	-5.43514	[.000]
CHECFA	0.	.395669E-17	0.	[1.00]
CHECGC	-.555112E-16	.103331E-16	-5.37219	[.000]
CHECGH	.277556E-16	.819181E-17	3.38821	[.001]
CHECGA	-.444089E-15	.338236E-16	-13.1296	[.000]

Wald Test for the Hypothesis that the given set of Parameters are jointly zero:

CHISQ(0) = 0.00000000 ; P-value = 1.00000

p=1, w=4.1, r=0.93, asset=100000,

Results of Parameter Analysis

Parameter	Estimate	Standard Error	t-statistic	P-value
EMU	96570.7	16452.5	5.86966	[.000]
EMUP	-17810.5	1089.49	-16.3476	[.000]
EMUW	1725.98	58.7034	29.4016	[.000]
EMUR	21277.4	15451.7	1.37703	[.169]
EMUA	.875167	.026905	32.5281	[.000]
ELMUP	-.184429	.032715	-5.63748	[.000]
ELMUW	.073278	.010473	6.99715	[.000]
ELMUR	.204906	.114119	1.79555	[.073]
ELMUA	.906245	.134753	6.72522	[.000]
ESTA	-88063.8	941.275	-93.5580	[.000]
ESTC	-25512.8	867.720	-29.4021	[.000]
ESTH	1807.82	25.8248	70.0033	[.000]
EFCP	-.193645	.028921	-6.69570	[.000]
EFCW	-.867867E-02	.201840E-02	-4.29977	[.000]
EFCR	0.	0.	0.	[1.00]
EFCM	.202323	.029828	6.78295	[.000]
EFHP	.029872	.680573E-02	4.38931	[.000]
EFHW	.061035	.931949E-02	6.54923	[.000]
EFHR	0.	0.	0.	[1.00]
EFHM	-.090908	.013661	-6.65443	[.000]
EFAP	0.	0.	0.	[1.00]
EFAW	0.	0.	0.	[1.00]
EFAR	-1.27608	.195162	-6.53855	[.000]

EFAM	1.27608	.195162	6.53855	[.000]
ELCP	-.230959	.029498	-7.82973	[.000]
ELCW	.614716E-02	.227719E-02	2.69944	[.007]
ELCR	.041457	.027225	1.52278	[.128]
ELCA	.183354	.022560	8.12735	[.000]
ELHP	.046639	.794140E-02	5.87284	[.000]
ELHW	.054374	.918312E-02	5.92107	[.000]
ELHR	-.018628	.012415	-1.50037	[.134]
ELHA	-.082385	.971942E-02	-8.47632	[.000]
ELAP	-.235346	.014996	-15.6940	[.000]
ELAW	.093508	.195102E-02	47.9278	[.000]
ELAR	-1.01460	.966909E-02	-104.932	[.000]
ELAA	1.15644	.014388	80.3745	[.000]
ELCPL	-.687708	.121472	-5.66144	[.000]
ELCWL	.168142	.023532	7.14512	[.000]
ELCRL	.129116	.089116	1.44885	[.147]
ELCAL	.390450	.049710	7.85458	[.000]
ELHPL	.251865	.046878	5.37277	[.000]
ELHWL	-.018414	.401277E-02	-4.58880	[.000]
ELHRL	-.058015	.040048	-1.44861	[.147]
ELHAL	-.175437	.020101	-8.72775	[.000]
ELAPL	-3.11612	.677215	-4.60137	[.000]
ELAWL	1.11523	.153419	7.26920	[.000]
ELARL	-.461725	.510114	-.905139	[.365]
ELAAL	2.46261	.231439	10.6405	[.000]

Wald Test for the Hypothesis that the given set of Parameters are jointly zero:

CHISQ(11) = 0.51795466E+17 ; P-value = 0.00000

p=1, w=4.1, r=0.93, asset=50000,

Results of Parameter Analysis

Parameter	Estimate	Standard Error	t-statistic	P-value
EMU	55571.1	14412.9	3.85566	[.000]
EMUP	-12981.6	2124.96	-6.10910	[.000]
EMUW	1520.56	151.911	10.0096	[.000]
EMUR	27276.5	12712.8	2.14560	[.032]
EMUA	.739025	.080914	9.13343	[.000]
ELMUP	-.233603	.042809	-5.45689	[.000]
ELMUW	.112186	.019200	5.84310	[.000]
ELMUR	.456481	.095446	4.78262	[.000]
ELMUA	.664936	.113356	5.86592	[.000]
ESTA	-37680.0	1077.12	-34.9822	[.000]
ESTC	-22730.1	967.397	-23.4961	[.000]
ESTH	1895.72	30.9583	61.2345	[.000]
EFCP	-.214648	.032713	-6.56162	[.000]
EFCW	-.012550	.380193E-02	-3.30096	[.001]
EFCR	0.	0.	0.	[1.00]
EFCM	.227198	.035760	6.35336	[.000]
EFHP	.036702	.010340	3.54967	[.000]
EFHW	.048655	.608567E-02	7.99508	[.000]
EFHR	0.	0.	0.	[1.00]
EFHM	-.085357	.013243	-6.44535	[.000]

EFAP	0.	0.	0.	[1.00]
EFAW	0.	0.	0.	[1.00]
EFAR	-1.97953	.382281	-5.17822	[.000]
EFAM	1.97953	.382281	5.17822	[.000]
ELCP	-.267722	.036647	-7.30544	[.000]
ELCW	.012938	.450675E-02	2.87091	[.004]
ELCR	.103712	.027190	3.81436	[.000]
ELCA	.151072	.031537	4.79027	[.000]
ELHP	.056642	.011932	4.74707	[.000]
ELHW	.039080	.643197E-02	6.07582	[.000]
ELHR	-.038964	.011445	-3.40431	[.001]
ELHA	-.056757	.989651E-02	-5.73506	[.000]
ELAP	-.462426	.051677	-8.94832	[.000]
ELAW	.222076	.774440E-02	28.6758	[.000]
ELAR	-1.07591	.021172	-50.8175	[.000]
ELAA	1.31626	.050940	25.8393	[.000]
ELCPL	-.664349	.116364	-5.70924	[.000]
ELCWL	.194850	.025529	7.63253	[.000]
ELCRL	.202148	.093679	2.15788	[.031]
ELCAL	.267350	.047725	5.60192	[.000]
ELHPL	.205652	.043185	4.76214	[.000]
ELHWL	-.029264	.778219E-02	-3.76037	[.000]
ELHRL	-.075946	.036766	-2.06569	[.039]
ELHAL	-.100442	.014970	-6.70956	[.000]
ELAPL	-3.91816	1.03515	-3.78511	[.000]
ELAWL	1.80704	.290053	6.23003	[.000]
ELARL	-.218257	.797013	-.273844	[.784]
ELAAL	2.32937	.230122	10.1223	[.000]

Wald Test for the Hypothesis that the given set of Parameters are jointly zero:

CHISQ(10) = 11587775. ; P-value = 0.00000

**** General Model *****

Full Information Maximum Likelihood

Equations: EQS1 EQS3 EQS4 EQS5 EQS6 EQS7

Endogenous variables: SS1 SS3 SS4 SS5 SS6 SS7

CONSTANTS:

	EPS	FC22	RC11	FC11	FC33
VALUE	0.00000	0.0100000	13034.90000	11.00000	0.020000

	RB23	FB23	RB13	FB13	RA33
VALUE	-9.00000	10.00000	-9.00000	10.00000	1.00000

	RA23	FA23	RA22	FA22	RA13
VALUE	0.00000	0.10000	0.00000	0.0100000	0.00000

```
                FA13        RA12        FA12        RA11        FA11
VALUE        0.10000     0.00000     0.10000     0.00000     1.00000


                 P89         P85         P86         P87         P88
VALUE        1.00000     0.87828     0.90955     0.92646     0.96027


                RR85        RB12        FB12        B111        B112
VALUE        1.00000    -9.00000    10.00000     0.00000     0.00000


                B113        B114        B116        B117        B118
VALUE        0.00000     0.00000     0.00000     0.00000     0.00000


                B119        AA13        AA23        AA12
VALUE        0.00000     1.00000     1.00000     1.00000

NOTE => The model is linear in the variables.
Working space used: 790671
                              STARTING VALUES

                RC12        RC13        RC22        RC23        RC33
VALUE       14.82860 76143.10000   350.53200 77353.30000     0.00000


                B330        B331        B332        B333        B334
VALUE  -877141.00000 -3989.56000  -367.03000     8.64496 24616.10000


                B335        B336        B337        B338        B339
VALUE    -1.41242D+06 -3489.06000  -98.33090     2.95763 -1470.74000


                  EW          ER          EA          ET        B220
VALUE      -0.020112    -1.01533 -1.58875D-06 -0.0098308   100.24200


                B221        B222        B223        B224        B225
VALUE      -35.61210    -1.91243    0.033636    11.01510 -1329.30000


                B226        B227        B228        B229        B110
VALUE      -17.17390    -0.60516   0.0030431    32.45600 -99979.30000


                B115
VALUE  -140612.00000

F= 29299.013308  FNEW= 29299.013336  ISQZ=  1 STEP= .500    CRIT= .29224E-03

CONVERGENCE ACHIEVED AFTER    1 ITERATIONS

    2 FUNCTION EVALUATIONS.


                      Full Information Maximum Likelihood
                      ===================================
```

Residual Covariance Matrix

	EQS1	EQS3	EQS4	EQS5
EQS1	4.76416D+08			
EQS3	-4.56084D+06	6712030.27479		
EQS4	-2.96178D+06	4753592.48018	6426082.01267	
EQS5	-5.30287D+06	3420360.86818	4189929.84373	5453723.10903
EQS6	-6.31126D+06	3209685.59132	3819082.93298	3988803.90874
EQS7	-871762.07567	2782714.89634	3639560.58034	3569635.27588

	EQS6	EQS7
EQS6	5180940.00117	
EQS7	4093739.19955	5651336.79955

Number of observations = 525 Log likelihood = -29299.0
Schwarz B.I.C. = 29419.8

Parameter	Estimate	Standard Error	t-statistic	P-value
RC12	14.8286	60.2521	.246109	[.806]
RC13	76143.1	17853.1	4.26498	[.000]
RC22	350.532	54.6282	6.41669	[.000]
RC23	77353.3	3645.78	21.2172	[.000]
RC33	0.	0.	0.	[1.00]
B330	-877141.	224388.	-3.90903	[.000]
B331	-3989.56	837.085	-4.76602	[.000]
B332	-367.030	56.7942	-6.46245	[.000]
B333	8.64496	1.57291	5.49614	[.000]
B334	24616.1	15050.4	1.63558	[.102]
B335	-.141242E+07	329417.	-4.28764	[.000]
B336	-3489.06	710.547	-4.91039	[.000]
B337	-98.3309	44.9346	-2.18831	[.029]
B338	2.95763	.923903	3.20124	[.001]
B339	-1470.74	7236.00	-.203253	[.839]
EW	-.020112	.438264E-02	-4.58902	[.000]
ER	-1.01533	.058141	-17.4632	[.000]
EA	-.158875E-05	.401148E-06	-3.96051	[.000]
ET	-.983078E-02	.327145E-02	-3.00502	[.003]
B220	100.242	467.995	.214195	[.830]
B221	-35.6121	1.99509	-17.8499	[.000]
B222	-1.91243	.160302	-11.9302	[.000]
B223	.033636	.544224E-02	6.18054	[.000]
B224	11.0151	11.5607	.952809	[.341]
B225	-1329.30	695.593	-1.91103	[.056]
B226	-17.1739	1.94799	-8.81623	[.000]
B227	-.605163	.121535	-4.97931	[.000]
B228	.304307E-02	.525861E-02	.578683	[.563]
B229	32.4560	15.6588	2.07271	[.038]
B110	-99979.3	20191.4	-4.95157	[.000]
B115	-140612.	29660.6	-4.74069	[.000]

Standard Errors computed from covariance of analytic first derivatives (BHHH)

Equation: EQS1
Dependent variable: SS1

Mean of dep. var. = 45686.9 Std. error of regression = 21827.3
Std. dev. of dep. var. = 52172.9 R-squared = .826701
Sum of squared residuals = .250127E+12 Durbin-Watson = 1.86521

```
   Variance of residuals = .476431E+09

Equation: EQS3
Dependent variable: SS3

       Mean of dep. var. = 6293.68      Std. error of regression = 2590.88
  Std. dev. of dep. var. = 7457.80                     R-squared = .884868
Sum of squared residuals = .352413E+10             Durbin-Watson = 1.93461
   Variance of residuals = .671263E+07

Equation: EQS4
Dependent variable: SS4

       Mean of dep. var. = 6038.36      Std. error of regression = 2535.12
  Std. dev. of dep. var. = 7566.20                     R-squared = .893293
Sum of squared residuals = .337409E+10             Durbin-Watson = 1.96202
   Variance of residuals = .642684E+07

Equation: EQS5
Dependent variable: SS5

       Mean of dep. var. = 5912.19      Std. error of regression = 2335.53
  Std. dev. of dep. var. = 7804.29                     R-squared = .918227
Sum of squared residuals = .286372E+10             Durbin-Watson = 2.09743
   Variance of residuals = .545470E+07

Equation: EQS6
Dependent variable: SS6

       Mean of dep. var. = 5740.44      Std. error of regression = 2276.34
  Std. dev. of dep. var. = 8271.85                     R-squared = .930023
Sum of squared residuals = .272039E+10             Durbin-Watson = 1.99442
   Variance of residuals = .518170E+07

Equation: EQS7
Dependent variable: SS7

       Mean of dep. var. = 5836.59      Std. error of regression = 2377.37
  Std. dev. of dep. var. = 9075.20                     R-squared = .932903
Sum of squared residuals = .296724E+10             Durbin-Watson = 1.88305
   Variance of residuals = .565188E+07

                             Univariate statistics
                             =====================

Number of Observations: 525
```

	Num.Obs	Mean	Std Dev	Minimum	Maximum
AAA	525.00000	-0.89197	0.32633	-2.42555	-0.32279
CCC	525.00000	1.32264D+10	6.30567D+09	3.85407D+09	4.52778D+10
AAACCC	525.00000	5.52318D+10	5.46555D+10	5.06208D+09	4.39294D+11
BBT	525.00000	-1.55705D+06	639877.01421	-4.77531D+06	-393694.62500
BB2T	525.00000	2.83306D+12	2.63200D+12	1.54995D+11	2.28035D+13
LAMT	525.00000	1564810.31708	642455.92815	397719.71875	4797142.50000
MUT	525.00000	-7764.14252	4047.67220	-37388.17969	-1971.54321
RANK1	525.00000	-1.55705D+06	639877.01421	-4.77531D+06	-393694.62500
RANK2	525.00000	1572574.46137	645049.94602	401744.81250	4818980.50000
PREDS1	525.00000	46794.56565	49532.52110	-34512.79688	284327.06250
ERRS1	525.00000	-1107.63326	21819.98009	-75923.17969	80649.85938
PREDS2	525.00000	61040.64341	32111.87734	8509.33008	174116.23438
ERRS2	525.00000	3427.21289	24704.32259	-85042.14844	122149.73438

PREDS3	525.00000	5795.12705	6745.36150	0.00000	27473.61328
ERRS3	525.00000	498.55147	2544.88058	-10335.99609	11582.76270
PREDS5	525.00000	5385.10298	7024.26368	-391.97012	40709.34766
ERRS5	525.00000	527.08483	2277.44649	-10815.19922	13047.36426
PREDS7	525.00000	5483.77877	8656.23148	-281.21466	67904.58594
ERRS7	525.00000	352.80727	2353.28574	-13888.41016	12824.55664

Results of Parameter Analysis

Parameter	Estimate	Standard Error	t-statistic	P-value
CHECKC	-.777156E-15	.286517E-14	-.271243	[.786]
CHECKH	.305311E-15	.936205E-15	.326116	[.744]
CHECKA	-.222045E-14	.100079E-13	-.221869	[.824]
CHECKM	1.000000	.952046E-15	.105037E+16	[.000]
CHECFC	0.	.182661E-15	0.	[1.00]
CHECFH	.111022E-15	.117595E-15	.944109	[.345]
CHECFA	0.	.733334E-15	0.	[1.00]
CHECGC	-.333067E-15	.251484E-14	-.132441	[.895]
CHECGH	.222045E-15	.100622E-14	.220673	[.825]
CHECGA	0.	.912295E-14	0.	[1.00]

Wald Test for the Hypothesis that the given set of Parameters are jointly zero:

CHISQ(0) = 0.00000000 ; P-value = 1.00000

p=1, w=4.1, r=0.93, asset=100000,

Results of Parameter Analysis

Parameter	Estimate	Standard Error	t-statistic	P-value
EMU	14964.6	4140.60	3.61410	[.000]
EMUP	817.922	139.463	5.86479	[.000]
EMUW	65.5787	25.2270	2.59954	[.009]
EMUR	13018.6	3377.15	3.85492	[.000]
EMUA	.017705	.878749E-02	2.01474	[.044]
ELMUP	.054657	.982644E-02	5.56227	[.000]
ELMUW	.017967	.250941E-02	7.15997	[.000]
ELMUR	.809066	.025281	32.0035	[.000]
ELMUA	.118310	.031472	3.75922	[.000]
ESTA	-86998.3	1151.07	-75.5805	[.000]
ESTC	-26890.4	1060.93	-25.3460	[.000]
ESTH	1902.15	29.1025	65.3604	[.000]
EFCP	-.422234	.119057	-3.54650	[.000]
EFCW	-.196938E-02	.803668E-02	-.245049	[.806]
EFCR	-2.29382	.666550	-3.44133	[.001]
EFCM	2.71802	.756084	3.59487	[.000]
EFHP	.679045E-02	.027674	.245369	[.806]
EFHW	.017730	.433523E-02	4.08972	[.000]
EFHR	.922780	.225962	4.08378	[.000]
EFHM	-.947301	.246152	-3.84844	[.000]
EFAP	-.762366	.220365	-3.45955	[.001]

EFAW	-.088948	.021993	-4.04441	[.000]
EFAR	-8.60088	2.27130	-3.78676	[.000]
EFAM	9.45220	2.51061	3.76491	[.000]
ELCP	-.273675	.079409	-3.44639	[.001]
ELCW	.046866	.690466E-02	6.78762	[.000]
ELCR	-.094760	.067147	-1.41123	[.158]
ELCA	.321569	.018827	17.0804	[.000]
ELHP	-.044986	.021727	-2.07049	[.038]
ELHW	.709463E-03	.362522E-02	.195702	[.845]
ELHR	.156352	.020200	7.74035	[.000]
ELHA	-.112075	.489156E-02	-22.9119	[.000]
ELAP	-.245735	.030214	-8.13326	[.000]
ELAW	.080883	.284135E-02	28.4664	[.000]
ELAR	-.953435	.021110	-45.1644	[.000]
ELAA	1.11829	.015451	72.3771	[.000]
ELCPL	-3.49614	.745413	-4.69021	[.000]
ELCWL	.270992	.029847	9.07931	[.000]
ELCRL	2.47175	.693095	3.56625	[.000]
ELCAL	.753395	.064742	11.6368	[.000]
ELHPL	1.07812	.222101	4.85422	[.000]
ELHWL	-.077404	.591969E-02	-13.0757	[.000]
ELHRL	-.738143	.214343	-3.44376	[.001]
ELHAL	-.262577	.017235	-15.2347	[.000]
ELAPL	-11.4522	2.32779	-4.91976	[.000]
ELAWL	.860303	.074059	11.6165	[.000]
ELARL	7.97187	2.22569	3.58176	[.000]
ELAAL	2.62000	.150560	17.4017	[.000]

Wald Test for the Hypothesis that the given set of Parameters are jointly zero:

CHISQ(12) = 0.44122857E+18 ; P-value = 0.00000

p=1, w=4.1, r=0.93, asset=50000,

Results of Parameter Analysis

Parameter	Estimate	Standard Error	t-statistic	P-value
EMU	14128.0	3770.52	3.74696	[.000]
EMUP	867.288	157.177	5.51790	[.000]
EMUW	61.9382	22.7550	2.72196	[.006]
EMUR	13135.7	3443.93	3.81416	[.000]
EMUA	.015811	.744228E-02	2.12446	[.034]
ELMUP	.061388	.797142E-02	7.70100	[.000]
ELMUW	.017975	.240275E-02	7.48089	[.000]
ELMUR	.864682	.010543	82.0129	[.000]
ELMUA	.055956	.014126	3.96124	[.000]
ESTA	-38358.8	1164.91	-32.9287	[.000]
ESTC	-22562.6	1073.54	-21.0169	[.000]
ESTH	2008.85	29.2107	68.7709	[.000]
EFCP	-.533023	.148528	-3.58870	[.000]
EFCW	-.248612E-02	.010119	-.245696	[.806]
EFCR	-2.89569	.807762	-3.58483	[.000]
EFCM	3.43120	.917086	3.74141	[.000]
EFHP	.681051E-02	.027689	.245960	[.806]
EFHW	.017782	.422074E-02	4.21307	[.000]

EFHR	.925507	.213224	4.34054	[.000]
EFHM	-.950100	.233069	-4.07647	[.000]
EFAP	-1.83144	.502827	-3.64229	[.000]
EFAW	-.213680	.049781	-4.29241	[.000]
EFAR	-20.6137	5.18522	-3.97547	[.000]
EFAM	22.6588	5.73003	3.95440	[.000]
ELCP	-.322389	.100609	-3.20437	[.001]
ELCW	.059189	.904297E-02	6.54527	[.000]
ELCR	.071206	.088030	.808882	[.419]
ELCA	.191995	.013327	14.4066	[.000]
ELHP	-.051514	.021726	-2.37112	[.018]
ELHW	.704525E-03	.363370E-02	.193887	[.846]
ELHR	.103973	.019739	5.26739	[.000]
ELHA	-.053163	.221537E-02	-23.9975	[.000]
ELAP	-.440463	.077363	-5.69349	[.000]
ELAW	.193606	.873974E-02	22.1524	[.000]
ELAR	-1.02103	.051920	-19.6655	[.000]
ELAA	1.26789	.039697	31.9394	[.000]
ELCPL	-4.11782	.887036	-4.64223	[.000]
ELCWL	.342122	.037857	9.03734	[.000]
ELCRL	3.31114	.843955	3.92336	[.000]
ELCAL	.464561	.043398	10.7047	[.000]
ELHPL	.999442	.206715	4.83489	[.000]
ELHWL	-.077640	.529388E-02	-14.6660	[.000]
ELHRL	-.793165	.203060	-3.90607	[.000]
ELHAL	-.128637	.833860E-02	-15.4267	[.000]
ELAPL	-25.5046	5.22246	-4.88363	[.000]
ELAWL	2.06203	.176262	11.6987	[.000]
ELARL	20.3747	5.07702	4.01312	[.000]
ELAAL	3.06785	.191440	16.0251	[.000]

Wald Test for the Hypothesis that the given set of Parameters are jointly zero:

CHISQ(12) = 0.39446272E+17 ; P-value = 0.00000

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END OF OUTPUT.

TOTAL NUMBER OF WARNING MESSAGES: 1918

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MEMORY USAGE:     ITEM:    DATA ARRAY  TOTAL MEMORY
                 UNITS:  (4-BYTE WORDS) (MEGABYTES)
MEMORY ALLOCATED         :   4500000       20.0
MEMORY ACTUALLY REQUIRED :   1083647        6.4
CURRENT VARIABLE STORAGE :    195167
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